Determinants of Board Size and Composition: A Theory of Corporate Boards

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DETERMINANTS OF BOARD SIZE AND COMPOSITION: A THEORY OF CORPORATE BOARDS

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Determinants of Board Size and Composition:
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ABSTRACT

I explicitly model the interaction of inside and outside corporate board members and address the question: What is the ideal size and composition of a corporate board? In my model the board is responsible for monitoring projects and making CEO succession decisions. Inside directors are better informed regarding the quality of firm investment projects proposed by the CEO, but outsiders can use CEO succession to motivate insiders to reveal their superior information. Board structure affects the flow of information and the effectiveness of the corporate board in replacing inferior projects with superior ones. The optimal board structure is determined by the tradeoff between maximizing the incentive for insiders to reveal their private information, minimizing the cost to outsiders to verify projects, and maximizing outsiders' ability to reject inferior projects. I also develop testable implications for the cross-sectional variations in the optimal board structure across firms.

The Determinants of Board Size and Composition:
A Theory of Corporate Boards

How do corporate boards function to monitor management and what is the ideal size and composition of a corporate board? This is a question that has been debated both among researchers and practitioners. Among practitioners, new regulations and practices such as the Sarbanes-Oxley Act have been proposed that restrict corporate board structure and seek to improve corporate governance. In academic literature, researchers have investigated many aspects of corporate boards, such as the relation between the proportion of outsiders on the board and firm performance. The existing evidence, however, has yielded mixed results about the types of board characteristics that facilitate effective monitoring and improved governance. Furthermore, there has been very little theory to explain board functioning or to provide insights into the anticipated effects of recent changes in regulations regarding corporate boards of directors. In this paper, I study aspects of the board functioning and I derive the optimal corporate board structure to monitor firms and maximize shareholder value. I also demonstrate how the optimal board size and composition and the effectiveness of the board as a monitoring mechanism vary with firm characteristics.

I develop a model of the functioning of the corporate board of directors and the interaction of the different members of the board with one another. I explicitly model the different types of board members (insiders and outsiders) and examine the way in which their numbers and proportions affect their interaction and the board's ability to monitor projects. In addition to highlighting the importance of both inside and outside members in monitoring, I endogenously derive the optimal board size and composition and the effectiveness of the board and show its dependence on firm and director characteristics. My paper addresses several questions on corporate board structure such as when are large size boards or boards with a higher proportion of insiders more effective.

My model is based on the idea that outsiders are independent of the CEO but, as many argue, they are relatively less informed about firm projects. Inside managers are an important source of firm-specific information, and their inclusion on the board can lead to a more effective decision-making process (Fama and Jensen (1983)). Adding insiders to the board also enables outsiders to better evaluate insiders for future positions (Hermalin and Weisbach (1988), Mace (1986)), which may increase the incentive for insiders to inform the board and win the approval of the outside board members.[1] At the same time, the incentives of inside board members may be distorted by private benefits and a possible lack of independence from the CEO. In terms of board size, smaller groups have the advantage of lower coordination costs and less free riding among members, but the disadvantages may include fewer people to inform the board.

The role of the corporate board in my model is twofold: (1) to monitor firm projects, and (2) to determine CEO succession. The CEO proposes a project to the board based on his incentives. Possible private benefits to firm managers may cause the CEO to propose an inferior project. Inside directors (in addition to the CEO) know whether or not the CEO proposed an inferior project, but outside directors can determine project quality only if they incur costly verification. To vote against a project, outside board members are required to verify that the

[1] In a survey of corporate directors, Mace (1986) documents that outside directors find insiders to be an important source of information to the board, enabling outsiders to directly ask managers without having to ask the CEO. In an experimental study, Gillete, Noe and Rebello (2003) find that boards comprised of informed but misaligned insiders and uninformed outsiders are able to implement institutionally preferred policies.

CEO proposed an inferior project. Verification costs to outsiders decrease if insiders reveal their private information. Outsiders use their CEO succession votes to motivate insiders to reveal their superior information, even though the incentives of insiders are distorted by private benefits from inferior projects. I use a Coalition-Proof Nash Equilibrium to endogenously solve for the decision of insiders on whether or not to inform the board. This allows insiders to decide individually as well as in credible coalitions.

The optimal board design maximizes the probability that the board will reject inferior projects. The board rejects an inferior project if a majority of the board votes against the project. Board size and composition affect the incentives of board members and play a crucial role in board effectiveness.

Given that insiders prefer to side with the CEO, a higher number of insiders increases the incentive for any one insider to inform outsiders because this may increase his chance of succession. However, a higher number of outsiders increases the cost to outsiders to coordinate their effort and verify projects. Therefore, small boards have the advantage of less coordination and more incentive for outsiders to verify projects, but less incentive for insiders to inform. On board composition, a higher proportion of insiders has the advantage of lower coordination costs for outsiders while still maintaining the high number of insiders. However, this requires more insiders to defect from the CEO for the board to be able to reject the proposed project. I also extend my model to allow the CEO to influence outside members and study how that affects optimal board structure and board effectiveness.

To date, there has been very little theory research which addresses board structure and the effectiveness of boards in monitoring firms. Hermalin and Weisbach (1998) examine the relationship between board of directors and the CEO and how that affects corporate board structure. Hermalin and Weisbach examine the endogenous dynamics of director nominations and CEO entrenchment, but I focus on endogenously deriving the board size and composition that maximizes the expected value of the firm. To the best of my knowledge, this is the first paper in the literature to derive optimal board size and board composition endogenously.

Warther (1998) considers how the CEO's ability to fire dissenting board members influences the decision-making ability of the board. Adams and Ferreira (2003) study the design of a board to provide incentives for the CEO to reveal his private information. My paper highlights the importance of other corporate insiders as an additional source of information to the board. I find that optimal boards have a higher proportion of insiders in firms where costs of project verification are high and firms where private benefits to insiders from inferior projects are low.[2]

In the empirical literature, papers have focused on finding particular board structures that may be more effective in monitoring than others (see surveys by John and Senbet (1998) and Hermalin and Weisbach (2003)). The evidence, however, is mixed. More recently, researchers have found cross-sectional differences on board size and composition (Denis and Sarin (1999), Gillan, Hartzell and Starks (2003), Lehn, Patro and Zhao (2003), Boone, Field, Karpoff and Raheja (2003)). The results in my paper explain some of the observed relationships between board structure, board effectiveness and firm characteristics, and provide new testable implications.

[2] In this paper, I specifically focus on boards as a monitoring mechanism. Others have examined how boards interact with the external takeover market (Hirshleifer and Thakor (1994)) or compared boards against other monitoring mechanisms (Maug (1997)). Almazan and Suarez (2003) study the allocation of power between the board and the other instruments available for aligning managers. By focusing solely on the board of directors, I can address the question of optimal board design for a given set of firm characteristics and what makes the board a more effective mechanism in one environment over another.

I find that firms where the incentives of insiders are better aligned with shareholders require smaller size boards. To the extent that the incentives are driven by insiders' low incentive to take inferior projects, firms that require smaller boards perform better. This result is consistent with empirical findings that firms with small size boards perform better (Yermack (1996), and Eisenberg, Sundgreen and Wells (1998)), but it also shows that large boards are optimal for some firms.

When I examine board composition, I find that firms where outsiders' incentives are relatively better aligned with the shareholders require a higher proportion of outsiders on the board. If the better incentives of outsiders are driven by better incentives to outsiders to verify projects, then the boards that require a higher proportion of outsiders perform better. This result helps explain why some papers in the literature (Hermalin and Weisbach (1991), Baysinger and Butler (1985) among others) have found that a higher proportion of outsiders on the board is associated with higher firm performance. If, however, the better incentives of outsiders are because the incentives of insiders are too inclined towards inferior projects, then firms that require a higher proportion of outsiders are less effective. This is consistent with Yermack (1996) and Agrawal and Knoeber (2001) among others, who report a negative correlation between the proportion of outside directors and Tobin's Q. Taken together, these results help explain why tests that do not condition on firm characteristics do not find a strong relation between the proportion of outsiders on the board and measures of firm performance. My results highlight that optimal board size and composition depend on firm and director characteristics.

The paper is organized as follows. Section II describes the project level agency problem and a board setup to monitor the firm. Section III analyzes the board and derives the optimal board size and composition and board effectiveness. Section IV extends the model to the case where the CEO influences outside board members. Section V analyses the comparative statics results that relate firm characteristics to the optimal board, and section VI concludes.

II. The Model

Consider a risk-neutral world with no discounting. The entrepreneur has access to a project with two possible (mutually exclusive) implementations. Both implementations require the same initial investment at time 0 and generate cash flows equal to X that are greater than the initial investment when the project succeeds (the good state) and cash flows equal to zero if the project fails (the bad state).

The implementations differ in their probability of a bad state. The probability of a bad state is normalized to zero for implementation 1 and it equals Φ for implementation 2. Φ is a continuous random variable distributed on (0,1]. I refer to implementation 1 as the good project and implementation 2 as the bad project, since implementation 2 has a higher probability of a bad state (I use the terms good and bad in a relative sense). Note that the higher the probability that the bad project will fail (larger Φ), the lower its net present value.

The model consists of three dates. The entrepreneur incorporates the firm at time 0 and hires a CEO and several managers to implement a project and a board to monitor the insiders and select a CEO successor. The firm has enough capital in its cash reserves to make the initial investment in a project. The CEO and corporate insiders observe the probability of failure of the bad project (Φ) at time 1 and based on his preference, the CEO proposes either the good project or the bad project to the board at time 1. At time 2 the firm invests either in the good project or the bad project. At time 3, Nature selects the final state and final cash flows from the project implemented are realized. The board selects a CEO successor at time 3 after the final cash flows are realized.

The CEO might obtain private benefits of control from the projects. A problem arises when the CEO receives higher private benefits from the bad project, causing the CEO to favor the bad project. These benefits are noncontractible and nonverifiable. They are in addition to the project cash flows and are available only to inside managers. The market value of the firm reflects only the project cash flows and does not include any part of the benefits of control, which can include managerial perks, human capital concerns, and effort aversion.

A. The Corporate Board

There are three types of players on the board, the CEO, the inside directors who are senior managers of the firm, and outside directors. All the players know that there is a good project and a bad project and that the CEO may propose the bad project. All board decisions are based on a simple majority vote.

I consider an extensive form game in which the board decides whether or not to approve the project proposed by the CEO. In the first stage of the model, outsiders solicit the proposed project's probability of a bad state from insiders, and each insider decides whether or not to inform the board. In the second stage, outside board members decide as a group whether to verify the proposed project or to implement it without verifying it.

1. Outsiders: The board includes at least one outside board member. All outside board members are identical and they benefit from high firm performance. Outsiders' motivation can be understood either as ownership of firm shares, which entitles directors to a share of the final value of the firm, or as benefits of reputation from the firm's good performance. Benefits of reputation mean that higher firm value will help outsiders to keep their current directorships and to obtain future directorships in other firms. For evidence on reputation benefits to directors, see Coles and Hoi (2003) and Ferris, Jagannathan and Pritchard (2003).[3]

Equation (1) captures the expected benefit from firm value to each outside board member. This benefit is at t=2 for a given project implementation, before the final outcome from the project is observed. Φ_p is the probability of failure of the proposed project. I note that Φ_p is equal to zero if the firm implements the good project, and equal to the observed probability of failure of the bad project if the firm implements the bad project. Therefore, the expected value of the firm is $(1-\Phi_p)X$. I assume that the expected benefit to outsiders is a linear function of the expected value of the firm, and that there are no benefits when firm value equals zero.

$$E(\text{Reputation Benefit}) = \mu\,(1-\Phi_p)X \qquad (1)$$

where μ measures the sensitivity of director's payoffs to firm value.

Outside members do not know which project the CEO proposed unless they incur costly verification. They decide as a group whether or not to verify the probability of a bad state of the proposed project before voting on it.[4] I first assume that all individual outsiders give identical votes both regarding the project and on the CEO succession voting rules. I later relax this assumption in section IV to allow the CEO to influence a proportion of the outside board members to vote in favor of his project even after the board verifies that the CEO proposed the bad project.

Outsiders are required to produce supporting information if they vote against the project, meaning that to vote against it, they must verify that the CEO proposed the bad project.

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[3] See also Gilson (1990), Kaplan and Reishus (1990), Brickley, Link and Coles (1999) and Harford (2000) for more evidence that directors benefit from high firm performance.

[4] Because my main intention is to study the strategic decision of inside members, I abstract somewhat from the strategic decisions of the outside members. By considering outsiders as a group, I ignore the strategic decisions of individual outsiders such as the incentive to free ride on the board.

This requirement can be thought out as the need for outsiders to have verifiable information to justifygoing against the CEO to shareholders or to any other third parties. Outsiders vote in favor of the project if they do not verify it or if they verify it and find that the CEO proposed the good project. Outsiders vote against the project if they verify that the CEO proposed the bad project.

Verification is costly, and outsiders need to coordinate their verification of the project. Outsiders solicit insiders' opinion on the probability of failure before they decide to verify the project. I assume that verification costs are large and it does not pay for outsiders to verify the project (given the ex-ante probability of the CEO proposing the bad project and the ex-ante probability of failure of the project) if no insider informs the board because it is difficult for outsiders to generate information when firm insiders have an incentive to conceal it.

If at least one insider reveals the true probability of a bad state of the proposed project, verification costs include a fixed cost of observing and producing the information, Ψ, that is incurred by each outside board member. This fixed verification cost depends on several factors, such as the industry membership of the firm and the amount of public information already available about the firm. For example, certain industries may be intrinsically more complex and therefore more difficult to evaluate than others. Thus, other things being equal, a grocery store chain may be easier to evaluate (lower Ψ) then a firm engaged in research for products in yet unproven technologies.[5]

In addition to the fixed verification cost, each outside board member incurs a coordination and communication cost, C, which increases with the number of outsiders on the board. This cost captures the difficulty in group decision-making as group size increases (Lipton and Lorsch (1992) and Jensen (1993)). For example, the increased cost of communicating and

[5] The fixed verification cost is similar to the one studied in Chemmanur and Fulgheri (1999), where they study the costs for outsiders in evaluating a firm in the IPO market.

coordinating the information with each other as the number of outsiders on the board increases, as well the greater difficulty in coming to a consensus on whether or not to verify a project. Carpenter and Westphal (2001) document that directors perceive to have a reduced influence on the board with increased board size, which may make it more difficult for board members to convince each other to monitor thus increasing the overall verification cost. See also Hackman (1990) for a study of the positive relation between the costs of group decision-making and group size. [6] m is the number of outsiders on the board. Ψ and C can be thought out as reduced form variables of the potentially more complicated difficulty in monitoring firm projects. Equation (2) shows the verification cost per outside board member when an insider reveals Φ_p:

$$\text{Verification Cost} = \Psi + C\,m \tag{2}$$

I note that the qualitative results of this model are robust to any other monotonically increasing cost function or when monitoring costs exhibit a threshold effect and increase only after some fixed m_0. In this case, the optimal minimum number of outsiders is the minimum number at which verification costs start to increase.

2. Insiders: There is at least one insider on the board, excluding the CEO.[7] Insiders are identical and they compete with each other to become the CEO's successor. Each insider has the same ex-ante probability of succeeding the CEO. Each insider decides whether to remain silent (support the proposed project), or inform the true probability of a bad state of the proposed project and vote against the project. Remaining silent is equivalent to telling the board that the CEO

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[6] A larger board size may also create an incentive problem similar to the free riding problem among diffuse shareholders (Berle and Means (1932) Demsetz and Lehn (1985)), making it more difficult for board members to convince each other to monitor. An alternative way to think about the coordination cost is that the reputation benefits are diminishing on the number of outsiders, decreasing their incentive to monitor.

[7] In a random sample of public firms, Denis and Sarin (1999) document that the mean fraction of inside directors is 0.39 (including the CEO), suggesting that insiders have a substantial fraction of the board seats.

proposed the good project. I assume that an insider votes against the proposed project if he informs the board that the CEO proposed the bad project.

The incentives of inside board members are distorted. Each insider can earn a private benefit equal to B from remaining silent if the CEO proposes the bad project and the board approves it. These private benefits can be thought out as managerial perks and effort aversion in the standard sense, but they can also measure insiders' resistance to speaking against the firm's CEO. For example, insiders may become more loyal towards the CEO as their tenure with the firm increases because of their personal relationship with the CEO.

The private benefits are in addition to the benefits that the CEO receives from the bad project and every insider who remains silent earns it, regardless of the number of insiders on the board. The insiders who inform the board do not receive any private benefit even if the bad project is accepted.

B. Succession Voting Rules

The entrepreneur sets the CEO succession voting rules to motivate insiders to inform the board and assist in the implementation of the good project. My assumptions about the succession votes are based on the idea that outside directors will select a successor independently of the current CEO only if they verify that the CEO proposed an inferior project (the board has lost confidence on the CEO) or if the firm ends up in a bad state. Otherwise, the board will go along with the CEO's successor choice. The CEO will select a successor from the set of insiders who supported him in his proposed project.[8]

[8] The evidence appears to be consistent with these assumptions. Although successful CEO's tend to influence the succession decision, corporate boards seem to choose their own successors when there are doubts on the CEO's performance. For example, in 1999 Coke replaced its CEO Ivester with a corporate insider, Daft. According to reports, Coke's board members lost confidence in Ivester and they believed that Daft would be better able to ensure the firm's performance. In 2000, Mattel replaced its chief executive with an outsider after incurring poor firm performance. Parrino (1997) documents that of the CEOs who are forced out, 49.6% are replaced by an outside executive. Of the CEO's who depart voluntarily, 90.1% are succeeded by insiders, suggesting that boards tend to

All outsiders vote for the same insider based on the voting rules. The CEO votes for an insider of his choice. Insiders do not vote on the succession since they are the ones being considered. Further, if there is only one outsider on the board, then the vote of the outsider prevails over the CEO's vote. Therefore, only outside board members' votes count in the succession decision.

I is the set of insiders who reveal the true project probability of a bad state and N-I is the set of insiders who remain silent. Before they vote on CEO succession, all players in the game find out which insiders belong to I and which insiders belong to N-I.

First, I examine the case in which outsiders verify the project probability of a bad state. In this case, outsiders randomly select one insider from the set I to receive all their votes, so that each insider in I has a 1/|I| chance of getting all the outside succession votes. |I| refers to the number of insiders in the set I. Outside board members select someone from outside the firm if I is empty. The insiders in N-I receive no succession votes from outsiders.

If the board does not verify the project, then they select a successor based on the final outcome of the project. If the final outcome is X (good outcome) all outsiders vote for the successor chosen by the current CEO. The CEO randomly selects a successor from the set N-I, so each insider in N-I has a 1/|N-I| chance of becoming the next CEO. |N-I| refers to the number of insiders in the set N-I. The CEO nominates someone from outside the firm if N-I is empty. The insiders in I receive no succession votes. If the final outcome is zero (bad outcome), the board selects an outsider to become the next CEO (no insider is selected). This last assumption is made for expositional clarity and it is consistent with shareholders advocating for new

select an insider when the CEO is performing well, but that insiders are also sometimes selected when the CEO is forced out.

management when the firm fails. The solution does not change if the board rewards insiders in I when the outcome of the project is zero.

Figure 1 describes a tree for this game.

I note that the requirement that outsiders verify a project and produce supporting information against a project before they can vote against it rules out the possibility that insiders will report a false probability of a bad state, and that the board will favor the bad project over the good project. To illustrate this, consider the case in which outsiders do not verify projects, but instead use the information to reject projects and reward the insiders who informed the board by selecting one of them to receive their succession votes. It is possible that in this scenario, some insiders would prefer to inform outsiders that the CEO proposed the bad project (and thus reveal a false probability of a bad state) even though the CEO proposed the good project. Doing so would benefit the revealing insiders by increasing their chances of becoming the CEO if the project succeeded. However, the information would be false and shareholders would be worse off since the board would end up implementing the bad project. This cannot happen if outsiders need to verify a project to vote against it.

III. Analysis of the Board

An equilibrium consists of (i) a decision of insiders to inform the board and (ii) a decision of outsiders to verify the proposed project. I study the stages of this problem backwards. First, outsiders maximize their expected payoffs by taking into account the information provided by insiders. Next, individual inside board members maximize their expected payoffs based on the expected response of the outsiders and the probability of the proposed project reaching the bad state. These results determine the optimal board structure.

A. Outside Board Members' Decision to Verify Projects

Let τ be the minimum number of insiders who must inform the board for outsiders to verify a project.

***Lemma 1**: t is greater than or equal to one, and t plus m outsiders constitute a majority on the board.*

Proofs of these and all subsequent results are given in the Appendix A.

Outsiders benefit from monitoring only if they can reject the proposed bad project, which happens only if τ insiders plus m outsiders have a majority on the board. Equation (3) shows the relation between the number of insiders (n) the number of outsiders (m) and τ. The CEO votes in favor of the proposed project. τ insiders plus m outsiders need to have one more person than the CEO plus the insiders who remain silent.

$$\tau + m = (1 + n - \tau) + 1 \tag{3}$$

In addition to the number of insiders that inform, outsiders take into account the verification costs and their expected benefits if they reject the proposed project and replace it with the alternative project. The benefits are the expected benefits of reputation if the firm implements the good project, minus the expected benefits if the firm implements the current project. Verification costs are as defined in equation (2).

***Proposition 1:** Outside board members verify the proposed project if and only if at least t insiders inform the board and if and only if outsiders believe that the probability of a bad state is greater than F_m. F_m is defined as:*

$$\Phi_m = \frac{Cm}{\mu X} + \frac{\Psi}{\mu X} \tag{4}$$

Φ_m is the minimum cutoff probability of a bad state that makes the expected benefits of verifying the proposed project larger than the costs.

B. Insiders' Decision to Reveal the Probability of Project Failure

Payoffs to inside board members depend on their utility of becoming the CEO, the private benefits from the bad project, and the probability of becoming the CEO. The probability of becoming CEO depends on whether or not the insider informs the board and on how many other insiders inform.

W(k) is the expected utility of an insider from informing the board, and S(k) is the expected utility of an insider from remaining silent given that the insider expects k other insiders to inform. All insiders remain silent when the CEO proposes the good project since they do not have an incentive to reveal a false project probability of a bad state. I focus on the case where the CEO proposes the bad project.

If an insider informs the board along with k other insiders, the insider has a 1/(k+1) chance of receiving the CEO succession votes if outsiders verify the project. The insider has no chance of receiving the votes if outsiders do not verify the project. If an insider remains silent while k insiders inform the board, the insider has a 1/(n-k) chance of receiving CEO succession votes if outsiders do not verify the project and the project succeeds. The insider has no chance of receiving the votes if outsiders verify the project. The insider who remains silent also earns a private benefit if outsiders do not verify the project.

The utility of receiving no promotion is normalized to zero. The utility of becoming the future CEO equals R, and the additional utility from the private benefit equals B. R is greater than B and both are greater than zero. Further, the total possible private benefit to insiders is less than the utility of an insider becoming the CEO (that is, $nB < R$).

$P(Ver)_k$ is the probability that outsiders verify the project, and $(1- P(Ver)_k)$ is the probability that outsiders do not verify the project, given that k insiders inform a probability of failure Φ_p to the board. As shown in the previous section, outsiders only verify a project if they expect to be able to reject it. Equations (5) and (6) define W(k) and S(k), respectively:

$$W(k) = \left(P(Ver)_{k+1} \left[\frac{R}{k+1} \right] \right) + \left(1 - P(Ver)_{k+1}\right) 0 \; = \; P(Ver)_{k+1} \left[\frac{R}{k+1} \right] \tag{5}$$

$$S(k) = P(Ver)_k (0) + (1\text{-}P(Ver)_k) \left((1-\Phi_p) \left[\frac{R}{n-k} \right] + (B) \right) =$$

$$= (1\text{-}P(Ver)_k) \left((1-\Phi_p) \left[\frac{R}{n-k} \right] + (B) \right) \tag{6}$$

In the simultaneous decision of insiders to inform the board, it cannot be an equilibrium to have some, but not all insiders inform. However, two pure Nash Equilibria are possible: either all insiders inform the board or no insider informs the board. All insiders inform the board if enough insiders are expected to inform for monitoring to take place, creating an incentive for everyone else to inform. No insider informs the board if enough insiders are expected to remain silent to prevent monitoring (Proof in Appendix A).

To select among the two possible solutions, I note that since insiders work together, it is realistic to allow insiders to communicate with each other to decide whether to reveal or to remain silent. However, since their agreements are not binding, any agreement between insiders must be credible in that it cannot pay for any insider or a subset of insiders to deviate further from the agreement.

The refinement of Nash equilibrium that allows players to decide individually as well as in credible coalitions is the Coalition Proof Nash Equilibrium (CPNE). In the CPNE, a given point is an equilibrium if there is no credible coalition of players that prefers to deviate from the equilibrium. Credibility requires that no insider or subset of insiders within the group benefits from deviating further from the group.

Proposition 2 (CPNE):

A) "All insiders inform" is the only CPNE if and only if $W(t\text{-}1) > S(k)$, $k=0,\ldots, t-1$.

B) "No insider informs" is the only CPNE if and only if $W(k)$ £ $S(t\text{-}1)$, $k=(t\text{-}1),t\ldots(n\text{-}1)$

If at least τ insiders always prefer to inform the board instead of remaining silent, then monitoring will take place. Therefore, all insiders inform the board and get a chance to receive the CEO succession votes. I note that in this equilibrium, the condition that no credible subgroup within τ prefers to deviate further and remain silent while k insiders inform is satisfied.

If there is a set of insiders that is large enough to prevent monitoring (so less then τ insiders are left informing) that prefers to remain silent regardless of the number of insiders who inform, then monitoring will not take place. Therefore, no insider informs the board. Again, there is no incentive for a subgroup to deviate further and inform the board to trigger monitoring.

Corollary 1*: "All insiders reveal" is the only CPNE if and only if $W(t\text{-}1) > S(t\text{-}1)$*

"No insiders reveal" is the only CPNE if the condition does not hold.

Intuitively, if outsiders verify the project, the insiders who inform share the probability of becoming the CEO successor. This means that a group of τ insiders benefits the most from informing the board because it has the least necessary number of people to inform the board and share the probability of becoming the CEO successor. If outsiders do not verify the project, only the insiders who remain silent get a chance to become the CEO. Each individual member in the group of τ insiders benefits the most from deviating alone and remaining silent so that (τ-1) insiders are left informing the board and out of the succession race. Therefore, it is sufficient for each member of a group of τ insiders to prefer to inform the board along with the other (τ-1) insiders for all insiders inform to be the CPNE. No insider informs the board is the CPNE if it pays for one of the insiders from the group of τ insiders to remain silent and prevent monitoring. Corollary 1 also implies that a unique CPNE always exists for this game.

Proposition 3: *All inside board members will inform the board if and only if the probability of the proposed project reaching the bad state is larger than the minimum necessary for outsiders to monitor and if and only if the probability of a bad state is larger than* Φ_n *where:*

$$\Phi_n = 1 - \frac{(R - B\tau)(n - \tau + 1)}{\tau R} \tag{7}$$

Φ_n is the minimum cutoff probability of a bad state that causes τ insiders to prefer to inform the board. The higher the probability of a bad state, the higher the incentive for τ insiders to inform the board.

C. **The Optimal Board**

Figure 2 shows the board's contribution to shareholder value in a firm where the CEO prefers to undertake the bad project.

Corollary 2: Φ_m, *the minimum cutoff probability of a bad state necessary for outsiders to verify the proposed project, increases with the number of outsiders on the board.*

The higher the number of outsiders on the board, the higher their coordination and communication costs and the lower their incentive to verify the proposed project.

***Corollary 3**: Φ_n, the minimum cutoff probability of a bad state necessary for t insiders to inform the board, decreases with the number of insiders on the board (n) and increases with t.*

If τ is constant, a higher number of insiders on the board increases the competition for the CEO succession votes and the incentive for a group of τ insiders to inform, decreasing the minimum probability of a bad state at which insiders are willing to inform the board. If the number of insiders on the board is constant, a higher τ (due to a lower number of outsiders) decreases the incentive for τ insiders to inform because each revealing insider gets a $1/\tau$ chance to receive the outsiders' succession votes if outsiders monitor.

***Lemma 2:** Let $Max[F_n,F_m] = F_{Max}$. The expected value of the firm increases as F_{Max} decreases.*

Proof: The board prevents the CEO from undertaking the bad project when the probability of a bad state is larger than both Φ_n and Φ_m. Since the probability of a bad state for the bad project ranges between zero and one, a lower Φ_{Max} implies a lower probability that the bad project will be implemented, and a higher average expected return from the bad projects that are implemented.

To find the optimal corporate board, I rewrite equation (3) for the number of outsiders (m) as a function of the number of insiders (n) and the number of insiders needed for a majority voting against the proposed project (τ). Equation (8) has the restriction that there be at least one outsider ($m \geq 1$) and at least one insider informs the board ($\tau \geq 1$).

$$m = n + 2 - 2\tau \qquad (8)$$

I first solve for the optimal number of insiders on the board ($n^*(\tau)$) as a function of the number of informants. Then I find the optimal number of informants (τ^*) that minimizes $\Phi_{max}(n^*(\tau), \tau)$.

Proposition 4: *For a given* t*, the optimal number of insiders on the board is at* $F_n = F_m$*.* $n^*(t)$ *is:*

$$n^*(\tau) = \frac{\tau R\left[1 + \frac{C(2\tau - 2) - \Psi}{\mu X}\right] + (B\tau - R)(1 - \tau)}{\frac{C\tau R}{\mu X} + R - \tau B} \tag{9}$$

Intuitively, when (τ) is constant, Φ_n decreases with the number of insiders and Φ_m increases with the number of insiders (a higher number of insiders implies a higher number of outsiders if τ is constant). The number of insiders that makes Φ_n equal to Φ_m minimizes Φ_{Max} for any given τ.

Proposition 5: *Let* $\hat{\tau}$ *be the value of* t *that makes* $\frac{dn^*(\tau)}{d\tau} = 2$. *If* $1 £ \hat{\tau} £ \frac{n+1}{2}$*, then* $\hat{\tau} = \tau^*$*, the optimal value of* t*. If* $\hat{\tau} < 1$, then $\tau^* = 1$. *If* $\hat{\tau} > \frac{n+1}{2}$, then $\tau^* = \frac{n+1}{2}$.

The formula for $\hat{\tau}$ is in Appendix A. Intuitively, equation (8) is the reason why $\frac{dn^*(\tau)}{d\tau} = 2$ is the optimal solution. When minimizing Φ_{Max}, I find the point where I cannot decrease the optimal solution for the number of outsiders by changing the number of insiders or the number of informants. Minimizing the optimal number of outsiders minimizes $\Phi_m(n^*(\tau), \tau^*)$ which in turn minimizes Φ_{Max} since Φ_n equal to Φ_m at the optimal solution for the number of insiders. The optimal number of informants equals $\hat{\tau}$ as long as the constraints hold that there be at least one outsider on the board and that at least one insider informs the board. As a last step, I find the optimal number of outside members by substituting the values of the optimal number of insiders ($n^*(\tau)$) and the optimal number of informants (τ^*) in equation (8).

IV. CEO Influence on Outside Board Members

So far, my model has examined a corporate board with no CEO influence over outside board members. Therefore, to be able to reject the proposed project in my model, it is sufficient for outsiders plus the defecting insiders to have a simple majority. This case may be the ideal, but in reality, shareholders have complained about CEO influence over board members. As the model of Hermalin and Weisbach (1998) shows, the CEO influence over the board becomes especially a concern as the CEO's tenure with the firm increases. Further, the new Sarbanes-Oxley Act and pending new rules at the stock exchanges are designed to ensure that board decisions become more independent of CEO influence, further indicating that CEO influence over board members is a concern to shareholders.

I can extend my model to allow the CEO to influence a proportion of outside board members to vote in favor of a bad project. I keep most of the assumptions in the model the same, with the only difference being that the CEO influences a proportion p of outside board members to vote in favor of his project after the board verifies that the CEO has proposed the bad project, where p is less than half of the outside members ($p < 0.5$). Since the current environment is one in which directors are influenced by the CEO, this extension makes the model more useful in determining optimal board structure. I also use the results from this extension to draw comparative static graphs to give a better sense of what might be expected in empirical work. Please refer to Appendix B for a detailed solution of the results.

The main effect of the CEO influence over outside board members is that the optimal proportion of outsiders on the board increases in most cases. The reason is that to reject the proposed project, outsiders now must overcome the votes of the outsiders influenced by the CEO, thus increasing the need for more outsiders on the board. The effectiveness of the optimal board decreases with the proportion of outsiders influenced by the CEO (Φ^*_{Max} $(n^*(\tau), \tau^*)$ increases with p). This solution suggests that the optimal number of outsiders on the board (and therefore, the board's size) should decrease as the new regulations become more effective in decreasing the CEO influence over the board members.

V. Comparative Statics

Firm and director characteristics affect the incentives of inside and outside directors causing the optimal board size and composition and the probability that the board will verify a bad project to vary on those characteristics. The parameters that I analyze are verification costs to outside board members and private benefits to insiders from inferior projects. The proofs for the comparative statics results are in Appendix C.

A. Verification Costs to Outsiders

Higher verification costs decrease the incentive for outsiders to verify projects (in terms of the model, Φ_m increases). Verification of projects can be relatively easy for firms that transact in markets characterized by stable prices and stable technology and for firms in industries with information that is more accessible to outsiders, for example, firms in mature or low technology industries with more tangible assets, and industries where it is easier to compare firms. In contrast, it may be harder for outsiders to verify projects in firms that specialize in new technologies, have a high R&D focus, and those that rely on specialized intangible assets. I note also that higher benefits of verification (either from outsiders' ownership of firm shares or in repute from the firm's market visibility) imply lower relative verification costs.

Figure 3 shows the optimal board size and composition at different levels of verification cost (Ψ) and figure 4 shows the minimum required probability of a bad state ($\Phi_{Max}(n^*, \tau^*)$) that is necessary for the board that is optimal at each level of verification cost to monitor (this measures the effectiveness of the board that is optimal for the given verification cost). These graphs allow the CEO to influence a proportion, p, of outside members. Proposition 6 formalizes the results.

Proposition 6 (changes in verification costs):

i) the optimal number of outsiders on the board decreases as fixed verification costs increase(i.e: $dm^*/d\Psi < 0$)*; ii) the board's dependence on the insiders' vote, as measured by* t*, and the proportion of insiders on the board, increases with verification costs for large values of verification costs (i.e:* $dt^*/d\Psi > 0$ and $d(n^*/m^*)/d\Psi > 0$ if $t^*>1$)*; and iii) the minimum required project failure rate for monitoring to take place is higher for optimal boards of firms with higher fixed verification costs (i.e:* $d\Phi_{Max}(n^*, \tau^*)/d\Psi > 0$).

If everything else is the same, them firms with higher verification costs to outsiders require a board structure with better incentives for outsiders to monitor projects, even though it comes at a cost of lower incentive for insiders to inform the board (decrease Φ_m even though this increases Φ_n so that Φ_m=Φ_n again). When verification costs are low, optimal boards have a large number of both insiders and outsiders and the minimum possible dependence on the insiders' vote (τ*=1). The large number of insiders gives them a high incentive to inform the board, while outsiders maintain the maximum voting power.

At low levels of verification costs (where τ*=1 constrains), the optimal number of outsiders and insiders decreases with verification costs, and board size decreases. The decrease in the number of outsiders lowers their coordination costs and offsets part of the higher verification costs, increasing the incentive for outsiders to verify projects while keeping their voting power. However, insiders' incentives to inform decrease since there are fewer insiders on the board.

At high levels of verification costs, I find that it is still optimal to decrease the number of outsiders as verification costs increase, but decreasing the number of insiders to maintain outside voting power provides less incentive for insiders to inform than allowing for a higher proportion of insiders on the board. As a result, optimal boards require a higher proportion of insiders in firms with high verification costs to outsiders. Even though this is the best solution, these boards require a higher minimum probability of a bad state to monitor projects (and therefore are less effective monitors) because of the low incentive for outsiders to verify projects.[9]

Denis and Sarin (1999) report that firms with greater growth opportunities have smaller boards with a smaller fraction of outsiders. Lasfer (2002) finds that high-growth British firms had a negative and significant change in firm value when they increased the number of outside board members due to the Cadbury Report recommendation. To the extent that high-growth captures project verification costs to outsiders, these two papers are consistent with my finding

[9] Since the optimal board in these firms requires more insiders, this result also suggests that firms with more costly project verification will benefit more from aligning the incentive of the managers with shareholders, such as increasing the pay for performance to insiders. Note also that empirically, some board members are officially outsiders, but they maintain close ties to the firm (gray board members) such as a venture capitalist firm. These board members may fill the required higher proportion of insiders in firms.

that it is better to have a relatively higher proportion of insiders on the board of firms with difficult-to –verify projects. Lasfer's result is also consistent with my finding that firms can be worse off by increasing the proportion of outsiders on the board if verification costs are high (non-optimal solution).

B. Private Benefits to Insiders

Private benefits measure the incentive of insiders to support the CEO in implementing inferior projects. Insiders' tenure in the firm, and the amount of free cash flow in the firm (Jensen (1986)) may measure private benefits. Further, private benefits may be lower with insiders' ownership of firm shares, since inferior projects decrease firm value, and industry competition, since insiders may fear that the firm will go out of business if the project fails. I note that the benefit to insiders from becoming the CEO (R), such as the increase in salary, also affect the incentive of insiders, but it has the opposite effect in that it increases the incentive for insiders to inform the board.

Figure 6 shows the optimal board size and composition for different levels of private benefits (B) and figure 7 shows the minimum required probability of a bad state (Φ_{Max}*(n*, τ*)) that is necessary for the board that is optimal at each level of private benefit (B) to monitor. These graphs allow the CEO to influence a proportion p of outside board members. Proposition 7 formalizes the results.

Proposition 7*:*

i) the optimal number of outsiders on the board increases as private benefits to insiders increase (i.e: dm*/dB > 0)*; ii) the board's dependence on the insiders' vote as measured by t, and the proportion of insiders on the board, decreases with private benefits up to the point where only* *one insider is needed to support outsiders in rejecting the proposed project. The board's dependence on the insiders does not increase with private benefits(i.e:* dt*/dB < 0 and d(n*/m*)/dB < 0 up until t*=1 constrains)*; iii) Optimal board size increases with private benefits at high levels of private benefits (i.e:* dm*/dB > 0 and dn*/dB > 0 when t*=1 constrains)*; and iv) the minimum required project failure rate for monitoring is higher for optimal boards of firms with higher private benefits (i.e:* $d\Phi_{Max}(n^*, \tau^*)/dB > 0$).

If everything else is the same, then firms with higher levels of private benefits to insiders require a board structure with better incentives for insiders to inform, even though it decreases the incentive for outsiders to verify projects (i.e: decrease Φ_n even though this increases Φ_m so that $\Phi_n=\Phi_m$ again). Optimal boards have a higher proportion of insiders and are more dependent on the insiders' vote ($\tau^*>1$) at low levels of private benefits. The higher proportion of insiders saves on the coordination costs of outsiders. Doing so also takes advantage of the better information of insiders and their high incentive to inform and assist in implementing superior projects. As private benefits to insiders increase, the number and proportion of outside board members increases, allowing outsiders to reject projects with less insider support (τ^* decreases).

If the optimal board requires only one insider to inform and assist outsiders in rejecting projects ($\tau^*=1$), then any further increase in private benefits requires a board with a higher number of both insiders and outsiders, thus increasing optimal board size. The larger number of insiders increases their incentive to inform the board, which offsets part of their higher private benefits. The larger number of outsiders maintains the outside voting power.

Therefore, optimal boards are larger at higher levels of private benefits. Even though a large board is the best solution, these boards are less effective in monitoring because there are higher coordination costs among outsiders. This result may help explain findings that firms with smaller boards have higher value (see Yermack (1996), and Eisenberg, Sundgren and Wells (1998), among others) when comparing across firms. At the same time, the results show that larger size boards are better within certain industries (see Adams and Mehran (2003) for bank firms).

C. Empirical Implications

Although the comparative statics focus on the partial effects of each exogenous variable (i.e., holding the other variables constant), it is important to note that the results should be considered jointly with the perspective of relative incentives of inside and outside board members. For example, this means that a company with high private benefits to insiders and high verification costs to outsiders may have a similar optimal board structure to a firm with low private benefits to insiders and low verification costs to outsiders. In terms of effectiveness, the board is most effective in firms with low project verification costs to outside board members, and in firms with less private benefits from inferior projects to insiders.

One prediction of my model is that there will be cross-sectional differences among different firm boards. Proxy statements provide anecdotal evidence that board size and composition should depend on the type of firm. For example, Microsoft has eight board members, with two insiders (including the CEO), one former firm executive, and one board founder and venture capitalist. General Electric has 17 board members, with four insiders and one former employee. I note that Microsoft, which is a high tech company, has a relatively smaller board with a higher proportion of insiders and affiliated outsiders. Due to the industry competition and the large ownership of the insiders, I would classify Microsoft as a difficult to verify firm with relatively aligned insiders. GE is a conglomerate. Because of the nature of the business, it is possible that potential private benefits to insiders be large in this company. Having a large number of insiders on the board provides more incentive for insiders to inform the board if the CEO proposes an inferior project. The outside-dominated board minimizes board dependence on the insider vote.

Another implication of my paper is that corporate board structure and managerial incentives will change as a firm moves in its life cycle. Verification costs will depend on the amount of public information available about the firm because this may help outsiders in evaluating firm projects. Thus, one can expect fixed verification costs to decrease as a firm becomes older and the firm's technology becomes better understood, causing the optimal proportion of outsiders on the board to increase and the board to become more effective. In terms of figure 3, the more mature firms would be on the left side of the graph where verification costs are low and the proportion of outsiders is high. In contrast, younger firms will better fit in the right side of the graph where verification of projects is difficult, boards require a higher proportion of insiders, and the overall board effectiveness is low. If we consider private benefits, it is possible that the younger and less mature firms have the least amount of private benefits (therefore, more insiders) due to the competitive nature of early state firms.[10]

VI. Conclusion

In this paper, I model the interaction of inside and outside board members and examine the way in which combinations of insiders and outsiders affect the monitoring effectiveness of corporate boards. I address the question of what determines board size and composition and how the optimal board structure varies with firm characteristics.

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[10] Anecdotal evidence about U.S. boards in the 90's seems to be consistent with these results. As it has been observed in the common press, firms in the Internet sector and industries with fast growing technology tend to have smaller boards with a higher proportion of insiders. According to a *Business Week* article, (Reingold (1999)) the "cutthroat environment" of these markets helps align the incentives of its insiders.

I consider a case where the CEO prefers an inferior project and how to set up a board that will maximize the probability of the board verifying and rejecting the inferior project. I note that once such a board is in place, it does not pay for the CEO to propose an inferior project since it would be rejected. This may explain why there are not common cases in the business press of insiders informing the board against the CEO, even though I show that their presence is important for the board to monitor the firm effectively.

Several extensions of this paper are possible. A natural extension is to consider the effects of CEO compensation on the optimal board structure. It can be shown in my model that boards become more effective monitors and they require a higher proportion of insiders as the managers' benefits from becoming the CEO increase. It would be instructive to learn about the optimal joint decision of CEO compensation and board structure.

A second extension is the effect of managerial competition on firm value. While having more insiders competing to become the future CEO may improve board performance, it is possible that in certain businesses a competition among insiders may be disruptive to the firm's business. A better understanding of the need to balance board effectiveness in monitoring with the ability of managers to work in improving firm value may help us also understand cases where firms may prefer to have a less than optimal board structure.

Finally, I note that the board structures I present maximize ex-ante shareholders' value. However, if board structures are designed according to the preferences of an already entrenched CEO, then they will not feature the board structure predicted in my analysis. Thus, when conflicts between managers and shareholders extend to the corporate board structure, the value of my analysis becomes primarily normative. In that case, my model shows how a move towards an optimal board structure may improve firm performance.

Figure 1: Board Decision and Payoffs Relating to Project Monitoring and CEO Succession

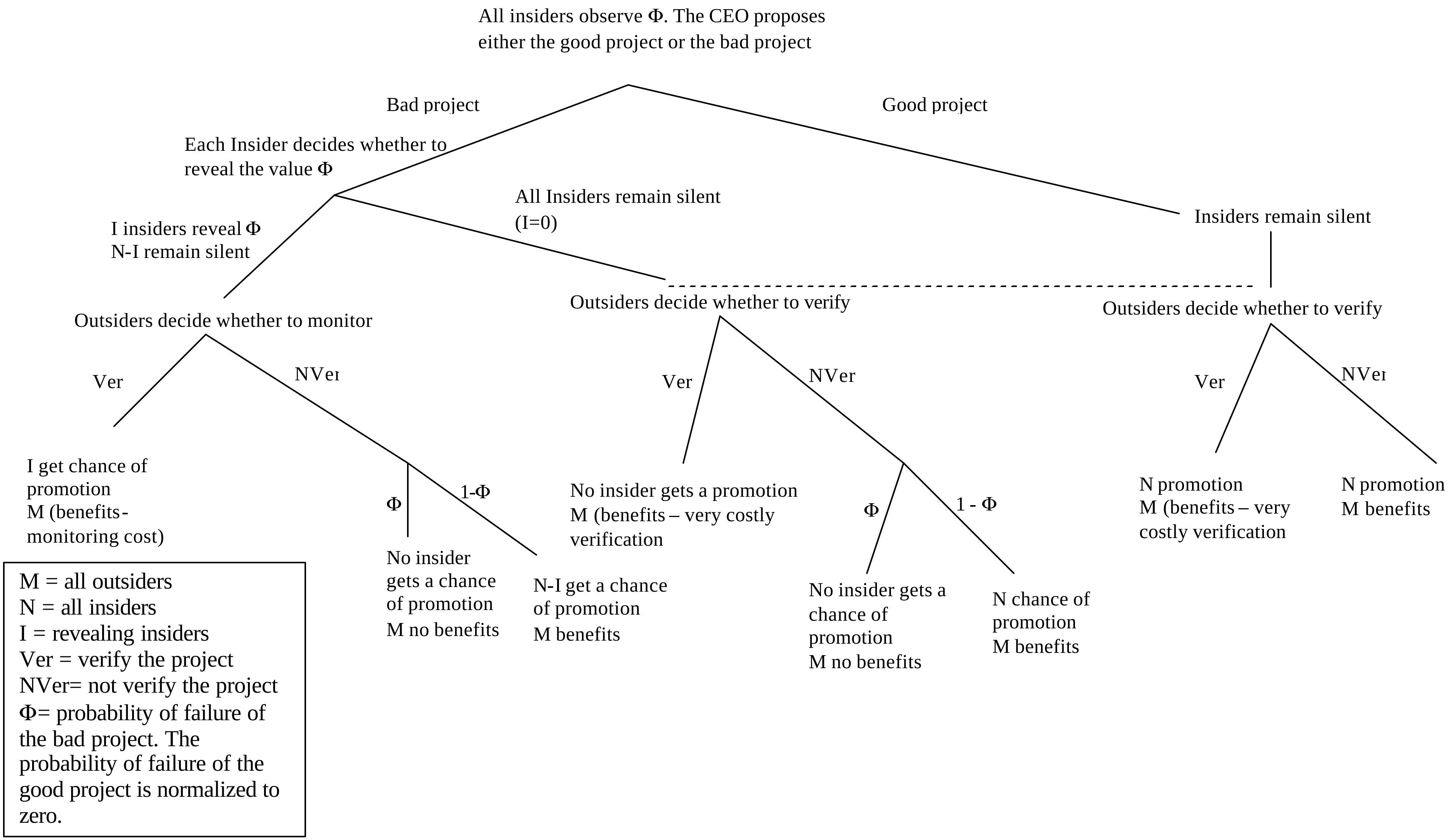
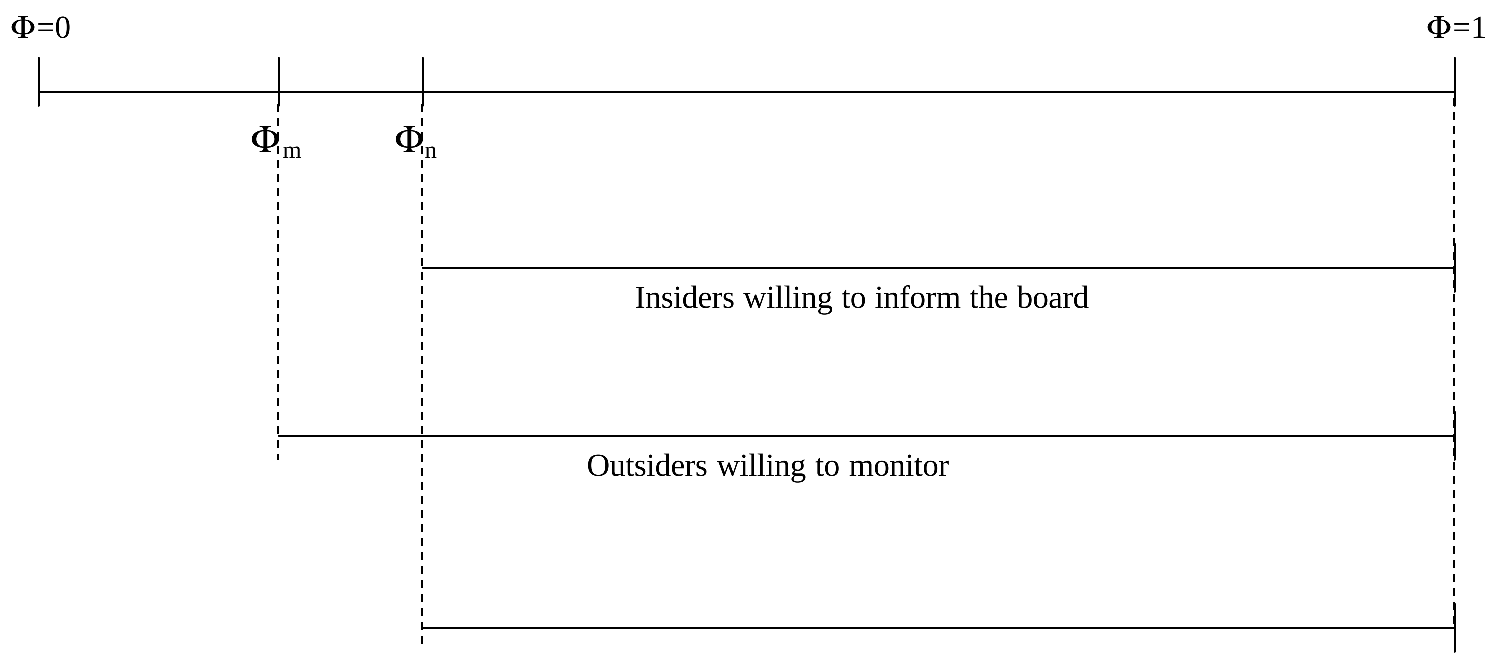


Figure 2: VALUE ADDED BY A BOARD WHEN THE CEO PREFERS THE BAD PROJECT

a. Φ is the probability that the bad project will reach a bad state. The higher the value of Φ, the lower the net present value of the project. Φ~U(0,1]

b. τ insiders will inform the board if the probability of the proposed project reaching a bad state is larger than Φ_n ($\Phi>\Phi_n$) and large enough for outsiders to monitor ($\Phi>\Phi_m$). Outsiders will verify the information if at least τ insiders inform the board and if the probability of a bad state is large enough for their expected benefits to be larger than their costs ($\Phi>\Phi_m$). Therefore, the board prevents the CEO from undertaking the bad project when the probability of bad state realization is larger than Max(Φ_n, Φ_m).

c. I note that the lower the Max(Φ_n, Φ_m), the greater the area in which the board intervenes. Therefore, ideally, we want to find a board that Minimizes [Max(Φ_n, Φ_m)]



Value added by the Corporate Board

Figure 3: Effect of Verification Costs on the Optimal Board Structure

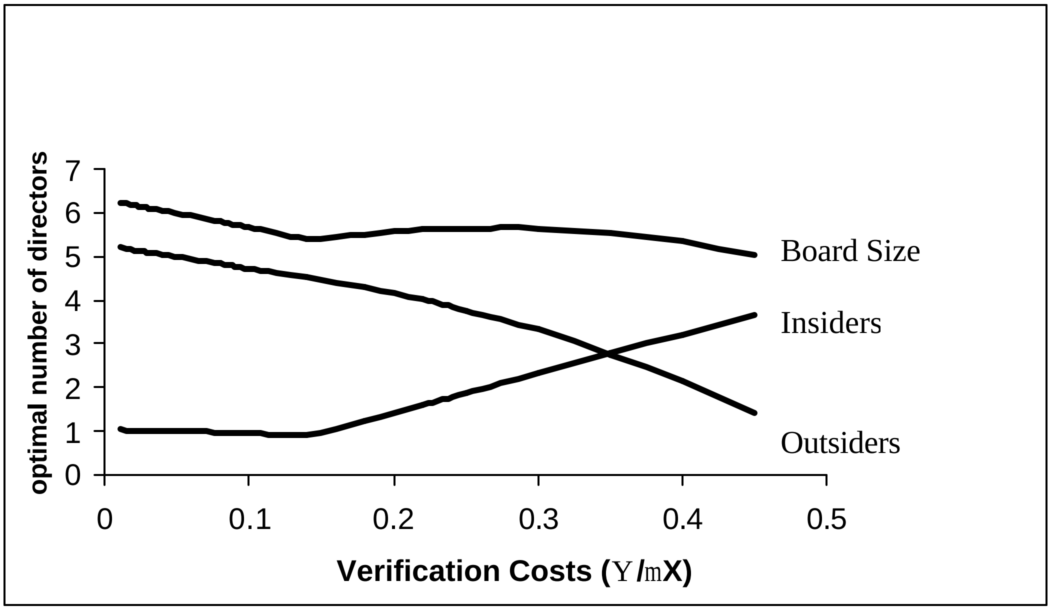


Figure 3: This figure illustrates changes in the optimal number of insiders (n*), the optimal number of outsiders (m*), and the optimal board size as the fixed costs of project verification change. In drawing the graph I assume that the utility of insiders from receiving a promotion in the firm is two units higher than the utility of not getting a promotion (R = 2). The private benefits to the insiders who go along with the bad project is equal to 0.2 if the project is implemented (B=0.2). The cost to outsiders in coordinating their actions is equal to 1% of their benefits of reputation from firm value (C/μX = 0.01). The CEO is able to influence 40% of the outside board members to vote in favor of his project choice (p = 0.4).

Figure 4: Verification Costs and

the Effectiveness of Optimal Board

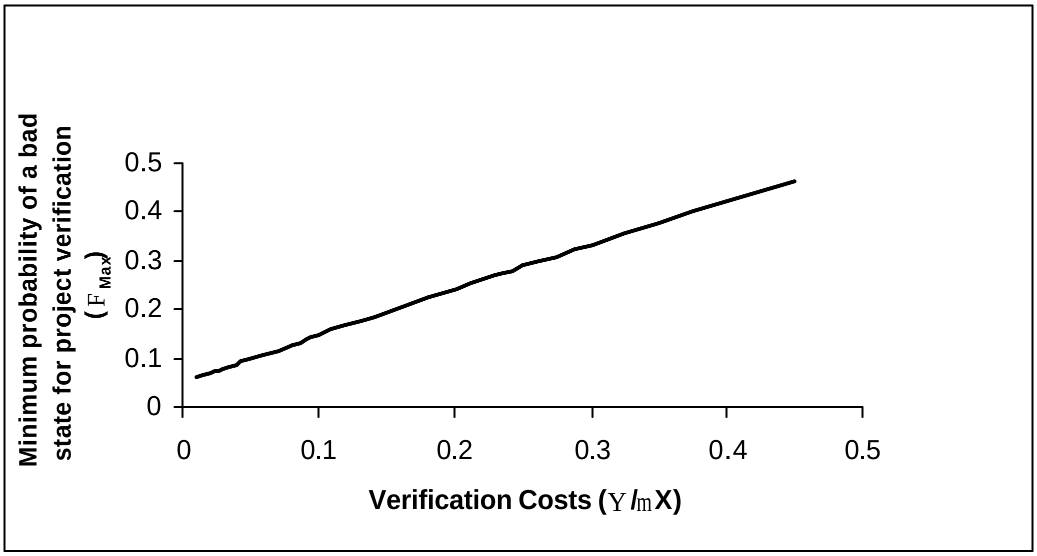


Figure 4: This figure illustrates how change in verification costs affects the optimal board's willingness to verify projects. Φ_{Max} is the minimum probability of a bad state for the optimal board to verify the project (that is, the board that is optimal for the given level of verification costs). A higher value of Φ_{Max} means that the optimal board requires a higher minimum probability of a bad state to verify projects.

In drawing the graph I assume that the utility of insiders from receiving a promotion in the firm is two units higher than the utility of not receiving a promotion (R = 2). The private benefits to the insiders who go along with the bad project is equal to 0.2 if the project is implemented (B=0.2). The cost to outsiders in coordinating their actions is equal to 1% of their benefits of reputation from firm value ($C/\mu X = 0.01$). The CEO is able to influence 40% of the outside board members to vote in favor of his project choice (p = 0.4).

Figure 5: Effects of Private Benefits on the Optimal Board Structure

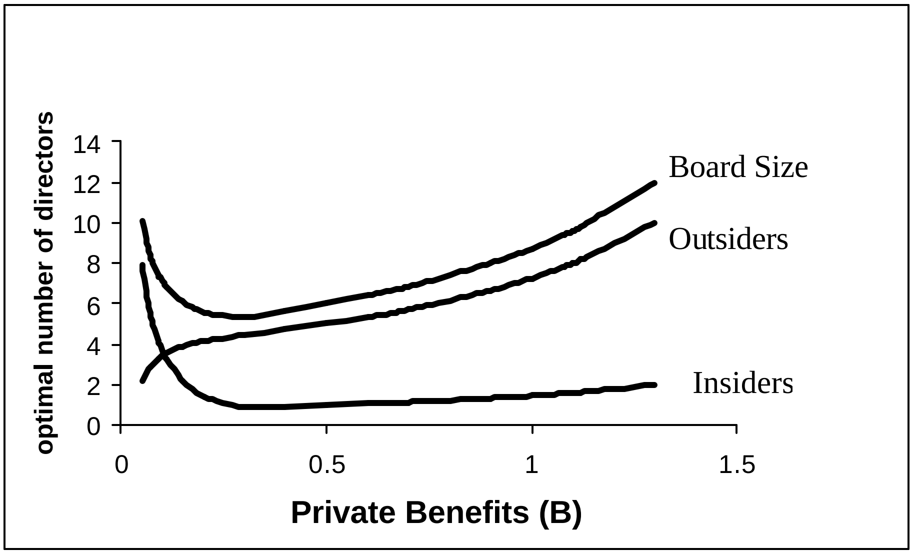


Figure 5: This figure illustrates how the optimal number of insiders (n*), the optimal number of outsiders (m*), and the optimal board size change with private benefits to inside board members. In drawing the graph, I assume that the utility of insiders from receiving a promotion in the firm is two units higher than the utility of not receiving a promotion (R = 2). The fixed cost of monitoring to outside board members equals 20% of their benefits of reputation from high firm value ($\Psi/\mu X=0.2$). The cost to outsiders in coordinating their actions is equal to 1% of their benefits of reputation from firm value ($C/\mu X = 0.01$). The CEO is able to influence 40% of the outside board members to vote in favor of his project choice (p = 0.4).

Figure 6: Private Benefits and
the Effectiveness of Optimal Board

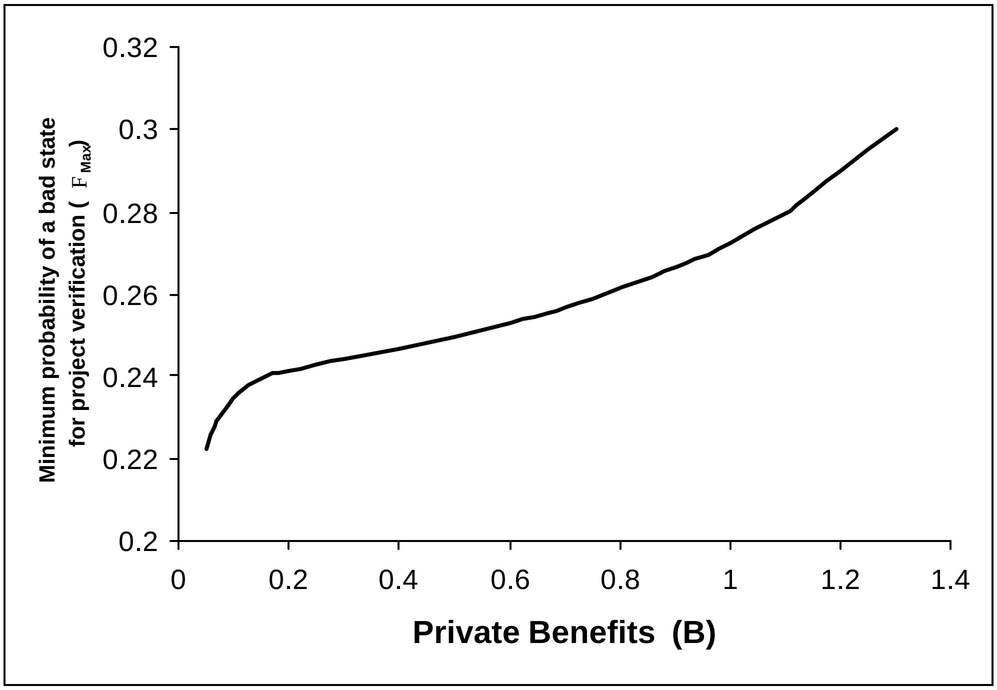


Figure 6: This figure illustrates how changes in private benefits to insiders affect the optimal board's willingness to verify projects. Φ_{Max} is the minimum probability of a bad state for the optimal board to verify the project. A higher value of Φ_{Max} means that the optimal board requires a higher minimum probability of a bad state to verify projects.

In drawing the graph, I assume that the utility of insiders from receiving a promotion in the firm is two units higher than the utility of not receiving a promotion (R = 2). The fixed cost of monitoring to outside board members equals 20% of their benefits of reputation from high firm value ($\Psi/\mu X$=0.2). The cost to outsiders in coordinating their actions is equal to 1% of their benefits of reputation from firm value ($C/\mu X$ = 0.01). The CEO is able to influence 40% of the outside board members to vote in favor of his project choice (p = 0.4).

References

Adams, R. B. and D. Ferreira 2003, "A Theory of Friendly Boards," Working Paper, Stockholm School of Economics

Adams, R. B and H. Mehran, 2003, "Is Corporate Governance Different for Bank Holding Companies?," *Economic Policy Review*, 9 (1), 123-142

Agrawal, A. and C. Knoeber, 2001. Do Some Outside Directors Play a Political Role? Journal of Law and Economics, 44(1), 179-198.

Almazan, A. and J. Suarez, 2003, "Entrenchment and Severance Pay in Optimal Governance Structures," *Journal of Finance*, 58 (2), 483-518.

Baysinger, B. and H. Butler, 1985, "Corporate Governance and the Board of Directors: Performance Effects of Changes in Board Composition," *Journal of Law, Economics, and Organizations* 1, 101-24.

Berle, A. D and G. C. Means. *The Modern Corporation and Private Property*. New York: Macmillan, 1932

Boone, A., L. Field, J. Karpoff, and C. Raheja, 2003, "The Evolution and Life Cycle of Corporate Boards: An Empirical Analysis" Working Paper, Penn State University.

Brickley, J., J. Link, and J. Coles, 1999, "What happens to CEOs after they retire? New evidence on career concerns, horizon problems, and CEO incentives," *Journal of Financial Economics*, 52, 341-377.

Carpenter, M. and J. D. Wesphal, 2001, "The Strategic Context of External Network Ties: Examining the Impact of Director Appointments on Board Involvement in Strategic Decision Making" *Academy of Management Journal*, 44, 639-660.

Chemmanur, T. and P. Fulgheri, 1999, "A Theory of the Going-Public Decision," *The Review of Financial Studies*, 12 (2), 249-279.

Coles, J. and C-K Hoi, 2003, "New Evidence of the Market for Directors: Board Membership and Pennsylvania Senate Bill 1310," *Journal of Finance*, 58 (1), 197-230

Demsetz, H. and K. Lehn, 1985, "The Structure of Corporate Ownership: Causes and Consequences," Journal of Political Economy 93, 1155-1177.

Denis, D. and A. Sarin 1999, "Ownership and Board Structure in Publicly Traded Corporations," *Journal of Financial Economics* 52, 187-223.

Eisenberg, T. S. Sundgren, and M. Wells, 1998, "Larger board size and decreasing firm value in small firms," *Journal of Financial Economics*, 48, 35-54.

Fama, E. and M. Jensen, 1983, "Separation of ownership and control," *Journal of Law and Economics*, 26, 301-325.

Ferris, S., Jagannathan M., and A.C. Pritchard, 2003, "Too busy to mind the business? Monitoring by directors with multiple board appointments," *Journal of Finance*, 58 (3), 1087-1112.

Gillan, S., Hartzell J., and L. Starks, 2003, "Industries, Investment Opportunities, and Corporate Governance Structures," Working Paper, University of Texas.

Gillete, A. B., Noe, T. H., and M. J. Rebello, 2003, "Corporate Board Composition, Protocols, and Voting Behavior: Experimental Evidence," *Journal of Finance*, 58 (5), 1997-2031.

Gilson, S., 1990, "Bankruptcy, boards, banks, and blockholders," *Journal of Financial Economics,* 27, 355-387.

Harford, J., 2000, "Takeover bids and target director's incentives: retention, experience and settling-up," Working Paper, University of Oregon.

Hackman, Richard, ed. *Groups that work* . San Francisco: Jossey-Bass, 1990.

Hermalin, B. and M. Weisbach, 1988, "The Determinants of Board Composition," The RAND Journal of Economics 19 (4), 589-606.

Hermalin, B. and M. Weisbach, 1991, "The effects of board composition and direct incentives on firm performance," *Financial Management*, 20 (4), 101-12.

Hermalin, B. and M. Weisbach, 1998, "Endogenously chosen boards of directors and their monitoring of the CEO," *American Economic Review*, 88, 96-118.

Hermalin, B. and M. Weisbach , 2003, "Board of Directors as an endogenously-determined institution: A survey of the economic literature," *Economic Policy Review*, 9 (1)

Hirshleifer, D. and A. Thakor, 1994, "Managerial performance, boards of directors and takeover bidding," *Journal of Corporate Finance,* 1, 63-90.

Jensen, M, 1986, "Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers," The American Economic Review 76, 323 –329.

Jensen, M. 1993. The Modern Industrial Revolution, Exit, and the Failure of Internal Control Systems. Journal of Finance 48 (3), 831-80.

John, K. and L. W. Senbet, 1998, "Corporate governance and board effectiveness," *Journal of Banking and Finance*, 22, 371-401.

Kaplan, S. and D. Reishus, 1990, "Outside directorships and corporate performance," *Journal of Financial Economics*, 27, 389-410.

Lasfer, M. A., 2002, "Board Structure and Agency Costs," Working Paper, City University Business School.

Lehn, K, and S. Patro and M. Zhao, 2003, "Determinants of the Size and Structure of Corporate Boards: 1935 -2000," Working Paper, University of Pittsburgh.

Lipton, K., and J. W. Lorsh, 1992, "A Modest Proposal for Improvement of Corporate Governance," Business Lawyer 48, 59-77.

Mace, M. L. 1986. Directors: Myth and Reality. Harvard Business School Press, Boston, MA

Maug, E., 1997, "Board of Directors and capital structure: alternative forms of corporate restructuring," *Journal of Corporate Finance*, 3, 113-139.

Parrino, R., 1997, "CEO turnover and outside succession, a cross-sectional analysis," *Journal of Financial Economics*, 46, 165-197.

Reingold, J, 1999, "Dot.com Boards are Flouting the Rules: They're small and packed with insiders. Does it matter?" *Business Week,* December 20, web site http:www.businessweek.com

Warther, V., 1998, "Board effectiveness and board dissent: A model of the board's relationship to management and shareholders," *Journal of Corporate Finance*, 4, 53-70.

Yermack, D., 1996 "Higher valuation of companies with a small Board of Directors," *Journal of Financial Economics,* 40, 185-212.

Appendix

Proofs:

***Lemma 1:*.**

There are no benefits to outsiders from verifying a project if all insiders remain silent, since either no insider revealed the truth (in which case monitoring costs are too large), or the CEO proposed the good project. Therefore, τ must be at least one. Further, outsiders benefit from monitoring only if they can reject the proposed bad project. τ insiders plus m outsiders need to be a majority for the board to reject the proposed project. Q.E.D.

Proposition 1:

Outside board members will verify the proposed project only if they expect their reputation benefits to be higher than the verification costs. The benefits are the expected benefits of reputation if the firm implements the good project (firm cash flows are expected to equal to X with probability 1), minus the expected benefits if the firm implements the current project (firm cash flows are expected to equal to X with probability ($1-\Phi_p$) and equal to zero with probability Φ_p):

$$\text{E(Reputation Benefit of Monitoring}/\Phi_p) = \mu X - \mu(1-\Phi_p)X = \mu\Phi_p X \tag{A.1}$$

Therefore, the following condition must be satisfied:

$$\mu\Phi X > \Psi + (C\,m)$$

$$\Phi > \frac{Cm}{\mu X} + \frac{\Psi}{\mu X} \tag{A.2}$$

Φ_m is the minimum cutoff such that (A.2) is satisfied.

Lemma 1 shows that at least τ insiders must inform the board for outsiders to verify a project. Q.E.D.

Proposition 2: Coalition Proof Nash Equilibrium of the decision of insiders to inform the board

Define:

W(k)= Utility of insider from revealing when k other insiders reveal

S(k)= Utility of insider from not revealing when k other insiders reveal

A. *Assumptions*:

ASSUMPTION 1: The incentives of τ insiders to inform the board affect the utility of the n insider.

(a) If at least t insiders reveal F, then the insider is better off revealing: reveal if k ³ t W(k) > S(k)

(b) If less then (t-2) insiders reveal, then the n individual insider is better off not revealing: not reveal if k£t-2 W(k) < S(k)

No assumption on k=t-1

ASSUMPTION 2: The decision to reveal depends on the utility of the insider number τ. BR is Best Response:

At k=t-1, if W(t-1)>S(t-1) then BR{i}= reveal.

if W(t-1)£S(t-1) then BR{i}= not to reveal.

ASSUMPTION 3: Each inside board member is better as the number of other insiders that may be selected to succeed the CEO decreases. This assumption implies that when $\Phi > \Phi_m$:

(a) The coalition that benefits the most from informing the board is a coalition of τ inside members

(b) The coalition of insiders that benefits the most from remaining silent is a coalition of ($n-\tau+1$) inside members, so that there are ($\tau-1$) insiders left informing the board

B. Proofs:

To derive the Coalition Proof Nash Equilibrium, it is necessary to first determine the Pure Nash Equilibrium points for this game. Only pure Nash Equilibrium are candidate points for Coalition Proof Nash Equilibrium:

There are exactly two pure strategy Nash Equilibria in this game:
i) Everyone reveals
ii) No one reveals

Proof: *Assumption 1 and Assumption 2 are required for the proof.*

Suppose k of N\{i} chose to reveal. What is i's best response (BR)?
If $k \le \tau-2$, BR{i}= not reveal
If $k \ge \tau$, BR{i}= reveal
If $k=\tau-1$, BR{i}= reveal if $W(\tau-1)>S(\tau-1)$ and all insiders will reveal or
BR{i}= not reveal if $W(\tau-1)\le S(\tau-1)$ and no insider will reveal

Therefore, it is clear that no one revealing and every one revealing are Nash Equilibria.

Are there other pure Nash Equilibria? No.
Suppose k reveal and (n-k) do not reveal
Case 1: $k\le\tau-2$, each agent in the revealing group is better off switching to the not revealing group
Case 2: $k \ge\tau$, each agent in the non revealing group is better of switching to revealing.
Case 3: if $k=\tau-1$
a) if $W(\tau-1)>S(\tau-1)$ then each agent in the non revealing group is better off revealing.
b) if $W(\tau-1)\le S(\tau-1)$ then each agent in the revealing group is better off not revealing.

Coalition Proof Nash Equilibrium:
Proof: *Assumption 1, Assumption 2 and Assumption 3 are required for the proof:*

The CPNE is the Nash equilibrium result that is stable against all possible self enforcing deviations. Two conditions need to be satisfied for a Nash Equilibrium to not be CPNE:
a) There exists a coalition where each member of the coalition is better off in the proposed deviation.
b) There is no further credible sub-coalition deviation that will make the sub-coalition better off (coalition needs to be credible).

Proof of part a: *All insiders revealing is the only CPNE if and only if $W(t-1)>S(k), k=0, ...,(t-1)$.*

Part I:
First show that no one revealing is not CPNE if $W(\tau-1)>S(k)$, $k=0, ...,\tau-1$.

Step1: Suppose no one reveals. Consider a coalition of $\{1,...,\tau\}\in A$.
Is each member of the coalition better off revealing?
If they deviate jointly,
Each $i \in A$: $W(\tau-1)>S(0)$, meaning that each member of the coalition prefers to inform the board.
Yes, there exists a coalition that would prefer to deviate from remaining silent.

Step2: Is the coalition credible?
If a sub-coalition $B\subset A$ deviates (back to not revealing Φ), will the sub-coalition be better off?
Based on the proposition, $W(\tau-1)>S(k)$, $k=0, ...,(\tau-1)$, which means that $S(\tau-|B|)<W(\tau-1)$

Therefore, members of the subcoalition cannot be made better off by deviating from informing the board, meaning that there exists a credible coalition of τ insiders that prefers to inform the board.
No one revealing is not CPNE.

Part II:
Second, show that all insiders revealing is CPNE. I demonstrate that no credible coalition will deviate from all insiders revealing.
Suppose every one reveals. Is there a coalition that prefers to deviate?
Deviating coalition has K members.
a) if $K < n - \tau + 1$
In this case $n\text{-}K > \tau\text{-}1$, Therefore, at least τ insiders inform the board and the coalitions does not have enough members to prevent monitoring.
In this case, the coalition is worse off remaining silent since assumption 1 states that an insider is better off revealing if at least τ insiders reveal Φ.
b) if $K > n - \tau + 1$
In this case, $n\text{-}k = \tau\text{-}1$ and the coalition has enough members to prevent monitoring. Is this coalition credible?
A coalition of K members in not credible because
$S(n\text{-}K)<W(\tau\text{-}1)$ since $(n\text{-}K)<\tau -1$ and the proposition states that $W(\tau\text{-}1) > S(k)$, $k=0, ...,(\tau\text{-}1)$
A subcoalition within K will be better off going back to revealing Φ.
Therefore, there is no credible deviating coalition that will prevent monitoring. All insiders revealing is a CPNE.

Part III:
All insiders revealing is the equilibrium only if the condition holds. Proof:
If $W(\tau\text{-}1) = S(k)$ $k=0, ..\tau\text{-}1$
Then a coalition of k members where $n\text{-}k = \tau\text{-}1$ will form to remain silent and prevent monitoring.
Coalition is large enough to prevent monitoring and the coalition is credible if
$W(\tau\text{-}1) = S(k)$ $k=0, ..\tau\text{-}1$

Therefore all insiders revealing will not be an equilibrium if the condition $W(\tau\text{-}1) > S(k)$, $k=0, ...,(\tau\text{-}1)$ does not hold.

Proof of part b*: No insider revealing is the only CPNE if and only if S(t-1)³W(k); k=t-1,...,(n-1).*
Part I:
Show that insiders revealing is not CPNE if $S(\tau\text{-}1) \geq W(k)$ $k=\tau\text{-}1, \ldots, n\text{-}1$

Step 1: Suppose all insiders reveal. Consider a coalition of $\{1,..K\} \in A$ formed to prevent monitoring.
If $n\text{-}K = \tau\text{-}1$ then the coalition has enough members to prevent monitoring. Suppose $K = n\text{-}\tau+1$ leaving $(\tau\text{-}1)$ revealing.
Is each member of the coalition better off remaining silent?
Each member of the coalition gets $S(\tau\text{-}1)$. According to the assumption, $S(\tau\text{-}1) \geq W(k)$ $k=\tau\text{-}1, \ldots, n\text{-}1$ which means that $S(\tau\text{-}1) \geq W(\tau\text{-}1)$ and the coalition would prefer to deviate from informing the board (remain silent)

Step 2: Is the coalition credible?

If sub-coalition $B \subseteq A$ deviates back to revealing, will the sub-coalition be better off?
Based on the proposition,

$$W(\tau-1+|B|)) \leq S(\tau-1)$$

Members of the subcoalition cannot be made better off by going back to revealing.
Therefore, there exists a credible coalition that prefers to remain silent and prevent monitoring.
All insiders revealing is not CPNE.

Part II:

Show that no insider revealing is CPNE. I demonstrate that no credible coalition will deviate from not revealing.
Suppose no one reveals. Is there a coalition that prefers to inform the board?
Deviating coalition has K members

If $K < \tau$, then the coalition does not have enough members to induce monitoring. In this case, the coalition is worse off informing the board, since assumption 1 states that all insiders are better off remaining silent if less then τ insiders inform the board.

If $K \geq \tau$ then the coalition has enough members to induce monitoring. Is the coalition credible?
A coalition of k members in not credible:
Each member of the coalition gets W(k) where $k=\tau-1,...,n-1$, since $K \geq \tau$.A further sub-coalition within K (of $(K-\tau+1)$ members) will further deviate (not reveal Φ), leaving τ-1 members to reveal:
The proposition states that $S(\tau-1) \geq W(k)$; $k=\tau-1,...,n-1$.
Therefore, there is no credible coalition that will inform the board. No insiders revealing is CPNE.

Part III:

No insiders revealing is the equilibrium only if the condition holds. Proof:
If $S(\tau-1) < W(k)$ $k=0, ..\tau-1$
Then a coalition of K members, where $K=\tau$ will form to inform the board. The coalition is large enough to induce monitoring and the coalition is credible if
$S(\tau-1) < W(\tau-1)$

Therefore, no insider revealing will not be an equilibrium if the condition $S(\tau-1) \geq W(k)$; $k=\tau-1,...,n-1$ does not hold.
Q.E.D.

Corollary 1:
Let k be the number of insiders who inform the board. I note that S(k) increases in k when outsiders do not monitor, because (n-k) insiders compete to become the CEO. Therefore, $S(\tau-1) > S(\tau-2) >\ldots> S(0)$ This means that it is sufficient for $W(\tau-1) > S(\tau-1)$ for condition (a) of proposition 2 to be satisfied (all insiders inform the board).

Similarly, W(k) decreases in k if outsiders monitor, because (k+1) insiders reveal and compete to become CEO. Therefore, $W(\tau-1) > W(\tau) \ldots> W(n-1)$, which means that it is sufficient for $W(\tau-1) = S(\tau-1)$ for condition (b) of proposition 2 to be satisfied (no insider informs the board). Q.E.D.

Proposition 3
First, Lemma 1A is necessary for the proof:

***Lemma 1A**: All insiders remain silent if the probability of the proposed project reaching a bad state is less than or equal to the minimum probability necessary for outsiders to verify the project. (i.e $F_p < F_m$)*

Based on the succession-voting rule, an insider who informs the board will not be promoted if outsiders do not verify the information. Therefore, the probability of failure of the proposed project needs to be greater than Φ_m for insiders to inform the board.

Second, corollary 1 states that insiders inform the board only if W(τ-1) > S(τ-1). The following equations derive the minimum cutoff necessary for insiders to inform the board (Φ_n). Note that $P(Ver)_\tau = 1$ and $P(Ver)_{\tau-1} = 0$ when $\Phi > \Phi_m$

$$W(k) = P(Ver)_{k+1}\left[\frac{R}{k+1}\right] \quad \rightarrow \quad W(\tau-1) = P(Ver)_\tau\left[\frac{R}{\tau}\right] = \left[\frac{R}{\tau}\right]$$

$$S(k) = (1\text{-}P(Ver)_k)\left((1-\Phi_p)\left[\frac{R}{n-k}\right]+(B)\right) \quad \rightarrow \quad S(\tau\text{-}1) = (1\text{-}P(Ver)_{\tau\text{-}1})\left((1-\Phi_p)\left[\frac{R}{n-(\tau+1)}\right]+(B)\right)$$

$$S(\tau-1) = (1-F)\left(\frac{R}{n-(\tau-1)}\right)+B$$

The condition for W(τ-1) > S(τ-1) is:

$$\frac{R}{\tau} > (1-\Phi)\left(\frac{R}{n-(\tau-1)}\right)+B$$

$$\Phi > 1-\frac{(R-B\tau)(n-\tau+1)}{\tau R} \qquad \text{(A.3)}$$

Φ_n is the minimum cutoff for which condition (A.3) is satisfied.
Q.E.D.

Corollary 2

Based on (A.2): $$\Phi_m = \frac{Cm}{\mu X} + \frac{\Psi}{\mu X} \qquad (4)$$

$$\frac{\partial \Phi_m}{\partial m} = \frac{C}{\mu X} > 0 \qquad \text{(A.4)}$$

The minimum cutoff Φ_m increases with the number of outsiders on the board, implying that outsiders are less willing to verify the project as their number on the board increases. Q.E.D.

Corollary 3:

Equation (7) in the text shows Φ_n: $$\Phi_n = 1-\frac{(R-B\tau)(n-\tau+1)}{\tau R} \qquad (7)$$

a) Proof that Φ_n decreases on n for a given τ:

$$\frac{\partial \Phi_n}{\partial n} = \frac{B}{R}-\frac{1}{\tau} \qquad \text{(A.5)}$$

The partial derivative of Φ_n with respect to n is negative if $\frac{B}{R}-\frac{1}{\tau}< 0$ or $B\tau < R$.
This condition holds since Bn < R and n > τ where n and τ are both greater than 0.

c) Proof that Φ_n increases on τ for a given n:

$$\frac{\partial \Phi_n}{\partial \tau} = \frac{(R-\tau B)\tau R + (n-\tau+1)B\tau R + R^2(n-\tau+1) - (n-\tau+1)B\tau R}{\tau^2 R^2} = \frac{R-\tau B}{\tau R} + \frac{(n-\tau+1)}{\tau^2} \qquad \text{(A.6)}$$

The partial derivative of Φ_n with respect to τ is positive since B<R/n, and n>τ. All variables are positive. Q.E.D.

Proposition 4

I show that Min[Max(Φ_n,Φ_m)] is at point where Φ_n=Φ_m

The relationship between inside and outside board members is as follows:

$m = n+2-2\tau \quad \rightarrow \quad \frac{\partial m}{\partial n} = 1$

Therefore, $\frac{\partial \Phi_m}{\partial n} = \frac{\partial \Phi_m}{\partial m}\frac{\partial m}{\partial n} = \frac{\partial \Phi_m}{\partial m}$

We have also seen that $\frac{\partial \Phi_n}{\partial n} < 0$ and $\frac{\partial \Phi_m}{\partial m} > 0$

If Φ_n >Φ_m, an increase in n will decrease Φ_n and cause Φ_m to increase. Since Max(Φ_n, Φ_m) is equal to Φ_n, the maximum will go down.

If Φ_m>Φ_n, a decrease in n will decrease Φ_m and increase Φ_n. Since Max(Φ_n, Φ_m) is equal to Φ_m, the maximum will go down.

Therefore, the Min[Max(Φ_n, Φ_m)] is found at the point where Φ_n =Φ_m. At this point, any changes in n will cause the Max(Φ_n, Φ_m) to go up. Q.E.D.

Finding n*(t):

Substitute $m = n + 2 - 2t$ in the equation for Φ_m:

$$\Phi_m = \frac{C\,(n+2-2t)}{\mu X} + \frac{\Psi}{\mu X} \qquad \text{(A.7)}$$

n* is the point where $\Phi_n = \Phi_m$:

$$\frac{C\,(n^*(\tau) + 2 - 2t)}{\mu X} + \frac{\Psi}{\mu X} = 1 - \frac{(R-B\tau)(n^*(\tau)-\tau+1)}{\tau R}$$

$$\tau R\left[\frac{C}{\mu X}(2-2\tau) + \frac{\Psi}{\mu X}\right] + \frac{C\tau R n^*(\tau)}{\mu X} = n^*(\tau)[B\tau - R] + [B\tau - R][1-\tau] + \tau R$$

$$n^*(\tau) = \frac{\tau R\left[1 + \frac{C(2\tau-2)-\Psi}{\mu X}\right] + (B\tau - R)(1-\tau)}{\frac{C\tau R}{\mu X} + R - \tau B} \qquad (9)$$

Proposition 5:

Given that $\Phi_n(n^*(\tau), \tau) = \Phi_m(n^*(\tau), \tau) = \Phi_{Max}(n^*(\tau), \tau)$, the solution for τ^* minimizes $\Phi_{Max}(n^*(\tau), \tau)$:

$$\underset{\tau}{\text{Min}}\ \Phi_{Max}(n^*(\tau),\tau) \rightarrow \underset{\tau}{\text{Min}}\ \frac{C}{\mu X}(n^*(\tau) + 2 - 2\tau) + \left(\frac{\Psi}{\mu X}\right)$$

subject to: $\tau^* \geq 1$

$n^*(\tau) + 2 - 2\tau^* \geq 1 \qquad (m \geq 1)$

If constraints are not binding:

FOC: $\frac{C}{mX}\left(\frac{dn^*(t)}{dt} - 2\right) = 0 \rightarrow \frac{dn^*(t)}{dt} = 2$

SOC: $\frac{C}{\mu X}\frac{\partial\left(\frac{dn^*(t)}{dt}\right)}{\partial\tau} > 0$, therefore if $\frac{\partial^2 n^*(\tau)}{\partial\tau^2} > 0$ then the solution for the FOC will be the minimum point.

The solution of the derivative of $n^*(\tau)$ with respect to τ is:

$$\frac{dn}{d\tau} = \frac{\tau^2\left[\frac{C}{\mu X}R - B\right]\left[\frac{C}{\mu X}2R - B\right] + 2\tau R\left[\frac{C}{\mu X}2R - B\right] + R^2\left[2 - \frac{C+\Psi}{\mu X}\right]}{\left[\tau\left(\frac{C}{\mu X}R - B\right) + R\right]^2} \qquad (A.8)$$

It can be shown that,

$$\frac{\partial^2 n}{\partial\tau^2} = \frac{2R^2\left[B\left(+1 - \frac{C+\Psi}{\mu X}\right) + \frac{CR(C+\Psi)}{(\mu X)^2}\right]}{\left(R - B\tau + \frac{CR\tau}{\mu X}\right)^3} \qquad (A.9)$$

I show that $\frac{\partial^2 n^*(\tau)}{\partial\tau^2} > 0$

Denominator:

$R - B\tau + \frac{CR\tau}{\mu X} > 0$ since $B\tau < R$

Nominator:

$2R^2\left[B\left(+1 - \frac{C+\Psi}{\mu X}\right) + \frac{CR(C+\Psi)}{(\mu X)^2}\right] > 0$ since

$2R^2 > 0$; $B - B\frac{C+\Psi}{\mu X} > 0$ because $\frac{\Psi + Cm}{\mu X} < \Phi_m$, which means that $\frac{C+\Psi}{\mu X} < 1$; and $\frac{CR(C+\Psi)}{(\mu X)^2} > 0$ since all variables are positive

Therefore, $\frac{\partial^2 n}{\partial \tau^2} > 0$. This means that the point where $\frac{dn^*(t)}{dt} = 2$ minimizes $\Phi_{Max}(n^*(\tau),\tau)$.
Q.E.D.

Solution for $\hat{\tau}$ and τ^*:

Let $\hat{\tau}$ be any positive value that satisfies $\frac{dn^*(\tau)}{d\tau} = 2$, $\hat{\tau}$ is the solution to the following equation:

$$\tau^2\left[\frac{C}{\mu X}R - B\right]\left[\frac{C}{\mu X}2R - B\right] + 2\tau R\left[\frac{C}{\mu X}2R - B\right] + R^2\left[2 - \frac{C+\Psi}{\mu X}\right] = 2\left[\tau^2\left(\frac{C}{\mu X}R - B\right)^2 + 2\tau\left(\frac{C}{\mu X}R - B\right)R + R^2\right]$$

Simplifying,

$$\tau^2 B\left[\frac{C}{\mu X}R - B\right] + 2\tau RB - R^2\left[\frac{C+\Psi}{\mu X}\right] = 0$$

$$\hat{\tau} = \frac{-b + \sqrt{(b)^2 - 4(a)(c)}}{2a} \qquad \text{(A.10)}$$

Where,

$a = \frac{B(CR)}{\mu X} - B^2$ $\qquad$ $b = 2BR$ $\qquad$ $c = -\frac{2R(C+\Psi)}{\mu X}$

Note that the variables a and b are positive and the variable c is negative. Therefore, $\sqrt{b^2 - 4ac} > b$. Only the positive root is a solution for $\hat{\tau}$

$\hat{\tau}$ becomes:

$$\hat{\tau} = \frac{R\left[\left(\sqrt{\left(\frac{C+\Psi}{\mu X}+1\right) + \frac{C}{\mu XB}\left(\frac{C+\Psi}{\mu X}\right)}\right) - 1\right]}{\frac{C}{\mu X}R - B} \qquad \text{(A.11)}$$

Solution for τ^*:

Constraint (1) is that $\tau^* = 1$. The Constraint (2) is that m = 1 or:
n +2 – 2 τ^* = 1 which means that $\tau^* \leq \frac{n+1}{2}$
Note that the above condition also guarantees that $n = \tau$ as long as n = 1.

Therefore,
If $1 \leq \hat{\tau} \leq \frac{n+1}{2}$, then $\hat{\tau} = \tau^*$, the optimal value of τ. If $\hat{\tau} < 1$, then $\tau^* = 1$. If $\hat{\tau} > \frac{n+1}{2}$, then $\tau^* = \frac{n+1}{2}$

Appendix B: CEO influence on outside board members

I keep most of the assumptions in the model the same. The costs of project verification remain the same. All outside board members vote according to the succession voting rules described in part 2 of section 1. The only difference is that the CEO influences a proportion p of outside board members to vote in favor of his project after the board verifies that the CEO has proposed the bad project, where p is less than half of the outside members ($p < 0.5$). All board members know that the CEO will be able to influence a proportion p of outsiders, and they account for this possibility when choosing their actions.

CEO influences a proportion p of outside members. I assume that the benefit to outsiders from the CEO influence is not large enough for outsiders to prefer to verify the proposed project when they don't expect to be able to reject it. Equation (3') defines the minimum number of insiders who must inform the board (τ_p) for monitoring to take place, accounting for the CEO influence over outsiders. The outsiders who are not influenced by the CEO plus the informing insiders need to have one more person than the CEO plus the insiders who remain silent. All variables have a subscript p to differentiate the case where the CEO influences outsiders:

$$\tau_p + (1-p)m_p = n_p + 1 - \tau_p + (p)m_p + 1 \qquad (3')$$

I rewrite this equation as a function of outsiders to solve for the optimal board:

$$m_p = \frac{n_p + 2 - 2\tau_p}{(1-2p)} \qquad (8')$$

Equation (9') has the restriction of at least one outsider ($m_p \geq 1$) and at least one insider to inform the board ($\tau_p \geq 1$). Note that this new equation for outsiders is similar to the previous equation (8) without the CEO influence, except that now I divide the equation by (1-2p). This has the effect of increasing the solution for the optimal number of outside members for a given number of insiders and minimum informants (τ_p).

I use equation (8') to substitute for the number of outsiders on the board. Note that a higher number of insiders still implies a higher number of outsiders for a given τ_p, so that proposition 4 does not change and the solution for the optimal number of insiders $n_p^*(\tau_p)$ is the point where $\Phi_n = \Phi_m$

Equation (9') shows the new optimal number of insiders on the board:

$$\frac{C\,(n_p(\tau_p)+2-2t_p)}{\mu X(1-2p)} + \frac{\Psi}{\mu X} = 1 - \frac{(R - B\tau_p)(n_p(\tau_p) - \tau_p + 1)}{\tau_p R}$$

$$n_p^*(\tau_p) = \frac{\tau_p R\left[1 + \frac{C(2\tau_p - 2)}{\mu X(1-2p)} - \frac{\Psi}{\mu X}\right] + \left(B\tau_p - R\right)(1-\tau_p)}{\frac{C\tau_p R}{\mu X(1-2p)} + R - \tau_p B} \qquad (9')$$

Solution for t_p*: τ_p^* is the point where $\Phi_{Max}(n_p^*(\tau_p), \tau_p)$ is minimized. Since the solution is the same as the case with no CEO influence, except that the coordination cost (C) is divided by (1-2p), I have omitted it in the interest of brevity. The solution is available upon request.

Let $\hat{\tau}_p$ be any positive value that satisfies $\frac{dn_p^*(\tau_p)}{d\tau_p} = 2$, $\hat{\tau}_p$ equals:

$$\hat{\tau}_p = \frac{-b + \sqrt{(b)^2 - 4(a)(c)}}{2a} \qquad (B.1)$$

Where,

$$a = \frac{BRC}{\mu X(1-2p)} - B^2 \qquad b = 2BR \qquad c = -\frac{R^2(C)}{\mu X(1-2p)} - \frac{R^2(\Psi)}{\mu X}$$

Note that the variables a and b are positive and the variable c is negative. Therefore, $\sqrt{b^2 - 4ac} > b$. Only the positive root is a solution for $\hat{\tau}$

Constraint (1) is that τ_p*=1.

Constraint (2) is that,

$$\frac{n_p{*}(\tau_p) + 2 - 2\tau_p{*}}{1-2p} \geq 1$$

That is:

$$\tau_p{*} \leq \frac{n_p + 1 + 2p}{2}$$

Note that this also guarantees that $\tau_p = n_p$ as long as $n_p = 1$

Proposition 5': $\hat{\tau}_p$ satisfies $\frac{dn_p^*(\tau)}{d\tau} = 2$. If $1 \leq \hat{\tau}_p \leq \frac{n+1+2p}{2}$, then $\hat{\tau}_p = \tau_p^*$, the optimal value of τ. If $\hat{\tau}_p < 1$, then $\tau_p^* = 1$. If $\hat{\tau}_p > \frac{n+1+2p}{2}$, then $\tau_p^* = \frac{n+1+2p}{2}$

We can now use equation (8') to substitute the optimal number of informants (τ_p^*), the optimal number of insiders $(n_p^*(\tau))$ to find the optimal number of outsiders on the board. The main effect of the CEO influence over outside board members is that the optimal proportion of outsiders on the board increases. The new solution for the optimal board is very similar to the case with no CEO influence, except that the communication and coordination cost (C) is divided by (1-2p). This increases the optimal number of outsiders for a $(n_p^*(\tau))$ and (τ_p^*).

Appendix C: Comparative Statics

Proposition 6: ***(****Changes in verification costs)*

All proofs allow the CEO to influence a proportion p of outside board members. These results are unchanged in the case where p=0 (the CEO does not influence outside board members). I skipped the subscript p to facilitate the exposure of the results.

i) Show that m* decreases with Ψ

$$m^* = \frac{1}{(1-2p)}\left(n^*(\tau) + 2 - 2\tau^*\right)$$

Step 1: Show that $\frac{dm^*}{d\Psi} = \frac{\partial n^*}{\partial \Psi}$, where $\frac{dn^*}{d\Psi} = \frac{\partial n^*}{\partial \Psi} + \frac{\partial n^*}{\partial \tau^*}\frac{d\tau^*}{d\Psi}$

$$\frac{dm^*}{d\Psi} = \frac{dn^*}{d\Psi} - 2\frac{d\tau^*}{d\Psi} = \frac{\partial n^*}{\partial \Psi} + \frac{\partial n^*}{\partial \tau^*}\frac{d\tau^*}{d\Psi} - 2\frac{d\tau^*}{d\Psi}$$

$$\frac{dm^*}{d\Psi} = \frac{\partial n^*}{\partial \Psi} + \left(\frac{\partial n^*}{\partial \tau^*} - 2\right)\frac{d\tau^*}{d\Psi} = \frac{\partial n^*}{\partial \Psi} \quad \text{since } \frac{\partial n^*}{\partial \tau^*} = 2$$

Step 2: Show that $\frac{\partial n^*}{\partial \Psi} < 0$ (I note that $\frac{\partial n^*}{\partial \Psi}$ is only the partial effect of a change in Ψ on n*(τ). The full effect on n of a change in Ψ needs to consider the change in τ* and how that affects n*(τ)

$$n^*(\tau) = \frac{\tau R\left[1 + \frac{C(2\tau - 2)}{\mu X(1-2p)} - \frac{\Psi}{\mu X}\right] + (B\tau - R)(1-\tau)}{\frac{C\tau R}{\mu X(1-2p)} + R - \tau B} \qquad (9')$$

$$\frac{\partial n^*}{\partial \Psi} = \frac{\tau R(-1)}{\left(\frac{C\tau R}{\mu X(1-2p)}\right)\mu X} < 0 \qquad (C.1)$$

That means that, $\frac{dm^*}{d\Psi} = \frac{\partial n^*}{\partial \Psi}$ < 0 and the optimal number of outsiders on the board decreases with verification costs when m*(τ) > 1.

ii) I show that τ* and the ratio of insiders to outsiders increases with Ψ.

I demonstrate that

$\frac{d\tau^*}{d\Psi} > 0$ in the region where τ*>1

Given $\hat{\tau}_p$, we know that,

$$w = \hat{\tau}^2\left[\frac{BRC}{\mu X(1-2p)} - B^2\right] + \hat{\tau}[2BR] - \frac{R^2(C)}{\mu X(1-2p)} - \frac{R^2(\Psi)}{\mu X} = 0 \qquad (C.2)$$

Solving implicitly,

$$\frac{d\hat{\tau}}{d\Psi} = -\frac{\frac{\partial w}{\partial \Psi}}{\frac{\partial w}{\partial \hat{\tau}}}$$

$$\frac{\partial w}{\partial \Psi} = -\frac{R^2}{\mu X} < 0$$

$$\frac{\partial w}{\partial \hat{\tau}} = 2\hat{\tau}B\left(\frac{C}{\mu X}R\right) - 2\hat{\tau}B^2 + 2BR$$

Show that $\frac{\partial w}{\partial \hat{\tau}}$ is positive since R>τB:

$$2\hat{\tau}B\left(\frac{C}{\mu X}R\right) - 2\hat{\tau}B^2 + 2BR > 2\hat{\tau}B\left(\frac{C}{\mu X}R\right) - 2BR + 2BR > 0$$

That means that $2\hat{\tau}B\left(\frac{C}{\mu X}R\right) - 2\hat{\tau}B^2 + 2BR > 0$

Therefore, $\frac{d\hat{\tau}}{d\Psi} = -\frac{\frac{\partial w}{\partial \Psi}}{\frac{\partial w}{\partial \hat{\tau}}} > 0$,

The solution implies that $\hat{\tau}$ increases as verification costs increase. If verification costs are very low, $\hat{\tau}$ <1, and therefore, $\tau^*=1$. An increase in verification cost will increase τ^* only after the point where $\hat{\tau}$ >1. Therefore, the voting power of inside board members increases with Ψ for large values of Ψ.

Increase in verification costs increase τ^*. An increase in τ^* increases the ratio of insiders to outsiders (n^*/m^*) since:

$$m^* = \frac{1}{(1-2p)}\left(n^*(\tau) + 2 - 2\tau^*\right)$$

$$n^* = 2\tau^* - 2 + m^*(1-2p) \quad \rightarrow \quad \frac{n^*}{m^*} = (1-2p) + \frac{2\tau^*-2}{m^*}$$

$$\frac{d\left(\frac{n^*}{m^*}\right)}{d\Psi} = \frac{2m^*\frac{d\tau^*}{d\Psi} - (2\tau^*-2)\frac{dm^*}{d\Psi}}{(m^*)^2} > 0 \quad \text{since} \quad \frac{d\tau^*}{d\Psi} > 0 \text{ and } \frac{dm^*}{d\Psi} < 0$$

and $2\tau^* - 2 > 0$ when $\tau^* > 1$

note that the ratio of n/m does not vary with Ψ when $\tau^*=1$

iii) I show that $\Phi^*_{Max}(n^*(\tau), \tau^*)$ increases with verification costs

A) Show that outsiders less willing monitor as verification costs increase

$$\frac{\partial \Phi_m}{\partial \Psi} = \frac{1}{\mu X} > 0$$

B) Show that $\Phi_{Max}(n^*(\tau), \tau^*)$ increases as verification costs increase.

Let $(n^*(\tau_0), \tau_0)$ be the optimal board solution for a given level of verification costs (Ψ_0).
At this point $[\Phi_m(n^*(\tau_0), \tau_0), \Psi_0)] = \Phi_n(n^*(\tau_0), \tau_0)/ \Psi_0$
Let $\Psi_1 > \Psi_0$. This causes:
$\Phi_m(n^*(\tau_0), \tau_0, \Psi_1) > \Phi_m(n^*(\tau_0), \tau_0, \Psi_0)$,
where $[\Phi_n(n^*(\tau_0), \tau_0)/ \Psi_1] = [\Phi_n(n^*(\tau_0), \tau_0)/ \Psi_0]$ because Φ_n does not depend on Ψ.

That means that given $(n^*(\tau_0), \tau_0)$, we have,
$\Phi_m(n^*(\tau_0), \tau_0, \Psi_1) > [\Phi_n(n^*(\tau_0), \tau_0)/ \Psi_1]$
The goal of the new corporate board structure is to $Min[Max(\Phi_n, \Phi_m)]$
We have seen that:

$$\frac{\partial \Phi_n}{\partial n} < 0, \ \frac{\partial \Phi_m}{\partial n} > 0, \ \frac{\partial \Phi_n}{\partial \tau} > 0 \text{ and } \frac{\partial \Phi_m}{\partial \tau} < 0$$

The new optimal corporate board structure $[n^*(\tau_1), \tau_1)]$ may decrease Φ_m so to decrease the $Max(\Phi_n, \Phi_m)$
However, any change in the corporate board structure (changes in $n^*(\tau)$ and τ) that would decrease Φ_m will have the effect of increasing Φ_n (they have an opposite effect on Φ_n).

Therefore, Max $[\Phi_n(n^*(\tau_1), \tau_1), \Phi_m(n^*(\tau_1), \tau_1, \Psi_1)] > \text{Max} [\Phi_n(n^*(\tau_0), \tau_0), \Phi_m(n^*(\tau_0), \tau_0, \Psi_0)]$.
Q.E.D.

Proposition 7: (Changes in private benefits):
All proofs allow the CEO to influence a proportion p of outside board members. These results are unchanged in the case where p=0 (the CEO does not influence outside board members). I skipped the subscript p to facilitate the exposure of the results.

i) Show that m* increases with B

where, $m^* = \frac{1}{(1-2p)}\left(n^*(\tau) + 2 - 2\tau^*\right)$

Step 1: Show that $\frac{dm^*}{dB} = \frac{\partial n^*}{\partial B}$

$$\frac{dm^*}{dB} = \frac{dn^*}{dB} - 2\frac{d\tau^*}{dB} = \frac{\partial n^*}{\partial B} + \frac{\partial n^*}{\partial \tau^*}\frac{d\tau^*}{dB} - 2\frac{d\tau^*}{dB}$$

$$\frac{dm^*}{dB} = \frac{\partial n^*}{\partial B} + \left(\frac{\partial n^*}{\partial \tau^*} - 2\right)\frac{d\tau^*}{dB} = \frac{\partial n^*}{\partial B} \quad \text{since } \frac{\partial n^*}{\partial \tau^*} = 2$$

Therefore, $\frac{dm}{dB} = \frac{\partial n}{\partial B}$

Step 2: Show that $\frac{\partial n^*}{\partial B} > 0$ (I note that when $\tau^*>1$, we cannot conclude that n* increases on B since $\frac{\partial n^*}{\partial B}$ does not include the effect of B on τ^*, and n* depends on τ^*)
.

$$\frac{\partial n^*(\tau)}{\partial B} = \frac{R\tau^2\left[1 + \frac{C\tau}{\mu X} - \frac{C+\Psi}{\mu X}\right]}{\left(R - B\tau + CR\tau\right)^2} > 0, \quad \text{since } \frac{C+\Psi}{\mu X} \leq 1 \tag{C.3}$$

That means, $\frac{dm^*}{dB} = \frac{\partial n^*}{\partial B} > 0$
Therefore, the optimal number of outsiders on the board increases with private benefits.

ii) Show that τ^* and the ratio of insiders to outsiders decreases with private benefits up until the point where $\tau = 1$ constrains.

A) I demonstrate that $\frac{d\tau^*}{dB} < 0$ when $\tau^* > 1$

Given $\hat{\tau}_p$, we know that,

$$w = \hat{\tau}^2\left[\frac{BRC}{\mu X(1-2p)} - B^2\right] + \hat{\tau}[2BR] - \frac{2R(C)}{\mu X(1-2p)} + \frac{2R(\Psi)}{\mu X} = 0 \tag{C.4}$$

Solving implicitly,

$$\frac{d\hat{\tau}}{dB} = -\frac{\frac{\partial w}{\partial B}}{\frac{\partial w}{\partial \hat{\tau}}}$$

$$\frac{\partial w}{\partial B} = \hat{\tau}^2 R \frac{C}{\mu X} - 2B\hat{\tau}^2 + 2\hat{\tau}R$$

I show that $\frac{\partial w}{\partial B}$ is positive (I use the condition R/n>B):

$$\hat{\tau}^2 R \frac{C}{\mu X} - 2B\hat{\tau}^2 + 2\hat{\tau}R > \hat{\tau}^2 R \frac{C}{\mu X} - 2R\hat{\tau} + 2\hat{\tau}R > 0$$

$$\frac{\partial w}{\partial \hat{\tau}} = 2\hat{\tau}B\left(\frac{C}{\mu X} R\right) - 2\hat{\tau}B^2 + 2BR$$

As previously shown in part (ii) of corollary 4, $\frac{\partial w}{\partial \hat{\tau}}$ is positive and therefore, $\frac{d\hat{\tau}}{dB} = -\frac{\frac{\partial w}{\partial B}}{\frac{\partial w}{\partial \hat{\tau}}} < 0$

The solution shows that $\hat{\tau}$ decreases with private benefits. If private benefits are low enough so that $\hat{\tau}>1$, then $\hat{\tau}=\tau^*$. An increase in private benefit will cause τ^* to decrease. However, when private benefits are large, $\hat{\tau}<1$ and $\tau^*=1$ constrains.

This also implies that the proportion of insiders on the board decreases with private benefits in the region where $\hat{\tau}>1$:

$$\frac{d\left(\frac{n^*}{m^*}\right)}{dB} = \frac{2m^*\frac{d\tau^*}{dB} - (2\tau^* - 2)\frac{dm^*}{dB}}{(m^*)^2} < 0 \quad \text{since} \quad \frac{d\tau^*}{dB} < 0 \text{ and } \frac{dm^*}{dB} > 0$$

$$\text{and} \quad 2\tau^* - 2 > 0 \quad \text{when} \quad \tau^* > 1$$

Increases in private benefits decrease τ^*, decreasing the ratio of insiders to outsiders on the board. The ratio is constant if $\tau^*=1$ constrains.

iii) Show that optimal board size increases when $\tau^*=1$ constrains

Part (ii) shows that $\hat{\tau}<1$, and $\tau^*=1$ at large levels of private benefits. In that case, $\frac{dn^*(\tau)}{dB} = \frac{\partial n^*(\tau)}{\partial B}$

As show in equation (C3), $\frac{\partial n^*(\tau)}{\partial B} > 0$

Part (i) of corollary 5 demonstrated that $\frac{dm^*}{dB} > 0$

Therefore, both the optimal number of insiders and the optimal number of outsiders on the board increase with private benefits. This means that board size increases with private benefits when private benefits are high (region where $\tau^*=1$ constrains) *iv)* Φ^*_{Max} $(n^*(\tau), \tau^*)$ is higher for optimal boards of firms with higher private benefits.

A) Show that insiders are less willing to inform as private benefits become large

$$\frac{\partial\Phi_n}{\partial B} = \frac{\tau R(n-\tau+1)\tau}{\tau R} > 0$$

B) Show that $\Phi_{Max}(n^*(\tau), \tau^*)$ increases as private benefits (B) increase.

Let $(n^*(\tau_0), \tau_0)$ be the optimal board solution for a given level of private benefit (B_0).
At this point $[\Phi_m(n^*(\tau_0), \tau_0)/ B_0] = \Phi_n(n^*(\tau_0), \tau_0, B_0)$
Let $B_1 > B_0$. This causes:
$\Phi_n(n^*(\tau_0), \tau_0, B_1) > \Phi_n(n^*(\tau_0), \tau_0, B_0)$,
where $[\Phi_m(n^*(\tau_0), \tau_0)/ B_1] = [\Phi_m(n^*(\tau_0), \tau_0)/ B_0]$ because Φ_m does not depend on B.

that means that given $(n^*(\tau_0), \tau_0)$, we have,
$\Phi_n(n^*(\tau_0), \tau_0, B_1) > [\Phi_m(n^*(\tau_0), \tau_0)/ B_1]$
The goal of the new corporate board structure is to $Min[Max(\Phi_n, \Phi_m)]$
We have seen that:
$\frac{\partial\Phi_n}{\partial n} < 0$, $\frac{\partial\Phi_m}{\partial n} > 0$, $\frac{\partial\Phi_n}{\partial \tau} > 0$ and $\frac{\partial\Phi_m}{\partial \tau} < 0$
The new optimal corporate board structure $[n^*(\tau_1), \tau_1)]$ will decrease Φ_n so as to decrease the $Max(\Phi_n, \Phi_m)$
Any change in the corporate board structure that would decrease Φ_n would also increase Φ_m.
Therefore, $Max [\Phi_n(n^*(\tau_1), \tau_1, B_1) , \Phi_m(n^*(\tau_1), \tau_1)] > Max [\Phi_n(n^*(\tau_0), \tau_0, B_0) , \Phi_m(n^*(\tau_0), \tau_0)]$.
Q.E.D.